AD-A120 488 MATHEMATICAL MODELLING OF THE BEHAVIOR OF THE LACOSTE AND ROMBERG 'G' GRA..(U) OHIO STATE UNIV COLUMBUS DEPT OF GEODETIC SCIENCE AND SURVEYI.. L A KRIEG NOV 81 321 1/2

UNCLASSIFIED AFGL-TR-81-0330 F19628-79-C-0075 F/G 8/5 NL

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MATHEMATICAL MODELLING OF THE BEHAVIOR OF THE LACOSTE AND ROMBERG "G" GRAVITY METER FOR USE IN GRAVITY NETWORK ADJUSTMENTS AND DATA ANALYSES

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THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION

NOVEMBER 1981

SCIENTIFIC REPORT NO. 10

AIR FORCE GEOPHYSICS LABORATORY
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
HANSCOM AFB, MASSACHUSETTS 01731

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFGL-TR-81-0330	2. GOVT ACCESSION NO.: AD-A120488
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): MATHEMATICAL MODELLING OF THE BEHAVIOR OF THE LACOSTE AND ROMBERG "G" GRAVITY METER FOR USE IN GRAVITY NETWORK ADJUSTMENTS AND DATA ANALYSES	5. TYPE OF REPORT & PERIOD COVERED: Scientific Report No. 10
	6. PERFORMING ORG. REPORT NUMBER: Report No. 321
7. AUTHOR(s): LENNY A. KRIEG	8. CONTRACT OR GRANT NUMBER(s): F19628-79-C-0075
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio 43210	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 62101F, 760003AL
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Geophysics Laboratory, Hanscom AFB, Massachusetts 01731, Contract Monitor - George Hadgigeorge/LW	12. REPORT DATE: November 1981
	13. NUMBER OF PAGES: 186 pages
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE: Unclassified

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Relative Gravity measurements
Gravity Meters

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report deals with the modelling of the behavior of the LaCoste Romberg "G" Gravity Meter. This is accomplished by using a model which includes a linear term plus a long wave sinusoidal term to represent the calibration information contained in the Calibration Table 1 supplied with each gravity meter. Additional sinusoidal terms are added to model any periodic screw error effects.

Using gravity meter data collected for the United States Gravity Base Station Network with the majority of the data being along the Mid-Continent

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Calibration Line, various models were tested. The control used were absolute gravity station values determined by the Italians, Marson and Alasia, and by Hammond of AFGL. The results indicate the presents of periodic screw error terms having a period of approximately 70.941 counter units with amplitudes less than 20 μgal and the estimated accuracy of gravity meter observations to be from 20-25 μgal. Due to the inconsistencies of the absolute gravity station determinations, the accuracy of the absolute values is probably closer to 20 μgal than the purported 10 μgal.

MICRO GAL

Foreword

This report was prepared by Mr. Lenny A. Krieg, Graduate Research Associate, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio, under Air Force Contract No. F19628-79-C-9975, The Ohio State University Research Foundation, Project No. 711715, Project Supervisor, Urho A. Uotila, Professor, Department of Geodetic Science and Surveying. The contract covering this research is administered by the Air Force Geophysics Laboratory (AFGL), Hanscom Air Force Base, Massachusetts, with Mr. George Hadgigeorge/LW, Contract Monitor.

Computer expenses were partially covered by the Instruction and Research Computer Center of The Ohio State University.

This report, or a modified version, will be submitted to the Graduate School of The Ohio State University as partial fulfillment of the requirements for the Doctor of Philosophy degree.

ACKNOWLEDGEMENTS

I would like express my gratitude and appreciation to all faculty, staff and fellow graduate students who gave me advice, help and just listened to me. Special thanks goes to my adviser, Dr. Urho A. Uotila, for his guidance and encouragement during the course of this work and my studies at the Department of Geodetic Science.

Additional thanks goes to the people at LaCoste & Romberg, Inc. in Austin, Texas, especially to Robert Perry and Lucien LaCoste, for their cooperation and hospitality during my visit to their facility.

My sincere thanks is extented to all fellow graudate students and friends who spent many hours listening to my problems and offered helpful advice, especially to Yehuda Bock, John Hannah, Erricos Pavlis, Christopher Jekeli and Mike Baker.

Dr. U. A. Uotila, Dr. R. H. Rapp and Dr. I. I. Mueller deserve especial thanks for serving on my reading committee and providing many constructive comments.

Finally, my deepest thanks goes to my family for their support during the course of my studies and to my wife, Anne, for her help in proofreading this paper.

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Studies in Statistics. Professors C. R. Rao, E. J. Dudewicz, P. R. Nelson and T. E. Obremski

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Chapter One

INTRODUCTION

1.1 Background Information

The International Gravity Standardization Net 1971 (IGSN 71) produced a world-wide gravity reference system of gravity base stations having a standard error for each station less than 0.1 mgal, 1 mgal = $10^{-5} m/s^2$, using absolute, pendulum, and gravity meter measurements [Morelli, et al., 1974]. See Table 1 for a summary of the measurements used in the IGSN 71 which indicates that the most widely used gravity meter in the IGSN 71 was the LaCoste & Romberg gravity meter. However, since the IGSN 71 results were adopted by the International Union of Geodesy and Geophysics (IUGG) in Moscow, 1971, and subsequently published [Morelli, et al., 1974], there has developed an ever increasing desire to obtain better values of gravity for gravity base stations.

Given an existing gravity base station network, the only way the standard error for the stations can be improved is by using new information. This new information can be in the form of either new measurements or improved modelling of the relationship between the observed quantities and the derived quantities.

Table 1 — Summary of data used in IGSN 71.

Instrument	Type of Instrument	Number of Instruments	Frequency of Use
ABSOLUTE	Cook	1	1 station
(10)	Sakuma	1	1 station
	Faller—Hammond	1	9 station
PENDULUM	Gulf	2	23 trips
(1200)	Cambridge	1	12 trips
	IGC	2	4 trips
	USCGS	2	2 trips
	DO	1	1 trip
	GSI	1	8 trips
GRAVITY METER	LaCoste—Romberg	53	98 trips
	Worden	14	12 trips
(24000)	Askania	2	6 trips
	North American	2	5 trips
	Western	3	2 trips

The quantities in parentheses represent the approximate number of measurements made with each class of instruments.

If new measurements are made, they can be classified as either absolute or relative in nature. Absolute measurements made with absolute measuring apparatus are used to determine the value of gravity which is the magnitude of the vertical gradient of the geopotential [Mueller and Rockie, 1966, pp 43-44], at a station by measuring the time it takes an object to fall a specific distance. The value of gravity obtained is the resultant of gravitation and the centrifugal force caused by the rotation of the earth [Mueller and Rockie, 1966, pp 43-44]. The physical dimension of gravity is an acceleration with its magnitude, in the geodetic community, given in either gal, mgal or μgal where 1 gal = 0.01 m/s^2, 1 gal = 1000 mgal, and 1 mgal = 1000 μgal. The value of gravity varies on the earth's surface from about 978 gal at the equator to about 983 gal at the poles [Heiskanen and Moritz, 1967, p 48].

Relative measurements are made using gravity meters. Gravity meters do not have the direct capability of measuring the absolute value of gravity but they are used to measure the difference in gravity between two stations. For gravity base station networks, the most commonly used gravity meter is the LaCoste & Romberg 'G' gravity meter manufactured by LaCoste & Romberg, Inc., Austin, Texas. This gravity meter was designed to be able to be used anywhere on the surface of the earth. Since the approximate gravity difference between the equator and the poles is about 5 gal, in order to insure world-wide measuring capabilites, the gravity meter has a measuring range of approximately 7 gal and is adjusted to work within the range of absolute gravity from

approximately 977 gal to 984 gal.

It is obvious that if the gravity values of a few stations were desired, and if time and funds presented no problem, the best method to obtain the stations' gravity values would be to make absolute gravity determinations at each station using either a transportable absolute gravity measuring apparatus or by installing a permanent absolute gravity measuring apparatus. However, due to restrictions on time and/or funds, this is not always feasible when the values of gravity at a large number of stations are desired. It takes approximately 2 to 4 days to set up and to make each absolute gravity determination. Thus, if only one absolute gravity measuring apparatus were used, it would take about 3 years to establish a gravity base station network of 300 stations, assuming the absolute apparatus worked continuously. To reduce the time required, gravity base station networks are established in which the gravity values of stations are resolved from relative gravity meter observations and the control is provided by a few stations whose gravity value has been determined by absolute gravity measuring apparatus.

The standard errors that can be associated with the gravity values of the stations in such a network depend, not only on the distribution and quality of the gravity meter observations and the absolute gravity value determinations made, but also on how well the relationship between the observed quantities and the derived quantites can be represented, i.e. mathematically modelled.

1.2 Description of Present Study

The nature of this study is to investigate the behavior of the LaCoste & Romberg 'G' gravity meter and develop a mathematical model which approximates this behavior. In order to test the mathematical models developed, data from the United Stated Gravity Base Station Network will be used. The majority of this data was obtained along the Mid-Continent Calibration Line which is located along the eastern side of the Rocky Mountains from New Mexico to Montana. Along this Mid-Continent Calibration Line, eleven absolute gravity stations were established at intervals of approximately 200 mgal to provide control. The vast majority of the gravity meter observations were made with a number of LaCoste & Romberg 'G' gravity meters with the remainder being made with two LaCoste & Romberg 'D' gravity meters, 'D17' and 'D43'. LaCoste & Romberg 'D' gravity meters work over a very limited gravity difference of approximately 200 mgal but can be adjusted to work anywhere in the range of the LaCoste & Romberg 'G' gravity meter.

In order to understand what causes the LaCoste & Romberg 'G' gravity meter to behave as it does, a review of the assembly and testing procedures used in its construction will be done. Most of the information about the gravity meter was based on first hand accounts obtained during a visit in December, 1980, to the LaCoste & Romberg, Inc. facilities in Austin, Texas.

From the information obtained about what goes into the construction and production of a 'G' gravity meter, attempts will be made to develop a more representative mathematical model of the instrument's behavior.

Another area to be explored is the method that could be used to indicate where new measurements, absolute or relative, should be made in order to improve any existing gravity base station network.

1.3 Review of Previous Studies

Various mathematical models have been proposed and used to model the behavior of the LaCoste & Romberg 'G' gravity meter [Morelli et al, 1974; Torge and Kanngieser, 1979]. There are basically two different types of models. One type of model uses as its observable the value in milligals of the observed gravity meter counter reading interpolated from the Calibration Table 1 supplied with each gravity meter [Uotila, 1974]; the other type of model uses the difference in value in milligal between two consecutive gravity meter observations as it observable [McConnell and Gantar, 1974; Whalen, 1974; Torge and Kanngieser, 1979]. Each model differs further in what parameters are included and how they are related. Some models include possible relationships for linear gravity meter drift [Uotila, 1974]. One model even postulates a relationship involving the square of stations' gravity values [Torge and Kanngieser, 1979].

One thing that is common for models proposed to date is that the Calibration Table 1 which is used to convert gravity meter counter readings to their values in milligal is assumed to be correct except for a linear scale factor which needs to be applied to all the values in milligal. Models have been proposed that include additional higher order scale factor terms [Uotila, 1974; Torge and Kanngieser, 1979].

Due to the construction of the LaCoste & Romberg 'G' gravity meter, there is a possibility of a periodic variation in the counter readings being introduced due to imperfections in the gear reduction system [Kiviniemi, 1974]. The amplitude of this effect has been estimated to be as large as 0.04 mgal [Torge and Kanngieser, 1979]. The attempts to solve for this periodic effect have resulted in no clear conclusion of its existence. One reason might be that the Calibration Table 1 is being used as a standard and assumed to be error free, thus masking the existence of the periodic effect. Another possibility is that the mathematical models proposed by Torge and Kanngieser [1979] are not be appropriate. Further, the control provide by the absolute gravity sites might not be distributed appropriately and be of high enough accuracy to solve for the periodic effects.

To better understand the requirements of the mathematical model needed to represent the behavior of the LaCoste & Romberg 'G' gravity meter, it is necessary to know how the gravity meter is constructed and how it works.

CHAPTER TWO

LACOSTE & ROMBERG "G" GRAVITY METER

2.1 Instrument's Development and History

The LaCoste & Romberg "G" gravity meter is a relative gravity measuring device designed to be used anywhere on the land surface of the earth. To achieve world-wide measuring capability, the "G" gravity meter must be able to measure differences in gravity as large as 5 gal. To satisfy this requirement, the "G" gravity meter was designed to be able to measure differences of approximately 7 gal and adjusted to work within the range of absolute gravity from approximately 977 gal to 984 gal. The smallest difference that can be determined directly from the "G" gravity meter observations corresponds to approximately 10 μgal.

The first LaCoste & Romberg land gravity meter was manufactured in 1939 and was the predecessor of the present "G" gravity meter. Compared to the present "G" models being manufactured, the first land gravity meter was much bigger and heavier. The current "G" gravity meters being manufactured and used are approximately 20 cm in length, 18 cm in width and 25 cm in height with each weighing approximately 3.2 kilograms excluding batteries [LaCoste & Romberg, 1981]. To date, in the neighborhood of 600 LaCoste & Romberg "G" gravity meters have

been built, with the current production level running about 30 to 40 meters per year. Each "G" gravity meter requires approximately 8 to 12 weeks to construct and test from the start of meter building until the gravity meter is delivered to the customer. However, due to the current large demand for the "G" gravity meter and the shortage of qualified gravity meter builders, the lead time is running around two years for the delivery of a "G" gravity meter [Perry, 1980, private communication].

2.2 Changes Made in "G" Gravity Meter

The actual internal workings of the "G" gravity meter have basically remained the same since the first one was manufactured in 1939. The major design changes that have occurred have been cosmetic to the meter case. The changes were made to reduce the size and weight of the gravity meter and to allow for electronic improvements and options such as electronic readout capability which is one of the options available on the "G" gravity meters. However, starting with meter 'G-458', a new gear reduction system (gear box) was installed into the "G" gravity meter. This gear box changes the gear ratios in addition to changing the type of gear box [Perry, 1980, private communication]. A detailed description of what this change affected is given in Section 2.4.2.

2.3 What Does a Gravity Meter Do?

An observation, O, made with a "G" gravity meter at a station is related to the absolute gravity value at the station by the following

relation:

$$f(O) + C + S = G \tag{2.1}$$

where

$f(O)$ — some functional relationship of the gravity meter's observation,

C — corrections that make the function $f(O)$ independent of the epoch of the observation,

S — an unknown offset value that must be added to obtain the correct absolute gravity value for the station,

G — absolute gravity value for the station.

Assuming that the unknown, S, is constant for an instrument during a particular time period, and the functional relationship, f(O), does not change during that time period, the gravity difference between two consecutive observations at station i and station j, can expressed by

$$f(O_i) - f(O_j) + C_i - C_j = G_i - G_j \tag{2.2}$$

where the subscript i refers to events at station i at one epoch and subscript j refers to events at station j at another epoch.

Eliminating the unknown offset, S, from equation (2.2) does not mean that the value of S is not needed. By this method, the value of the unknown offset, S, remains unknown but does not enter directly into equation (2.2). However, the value of S is still needed, as can be seen in equation (2.1), to determine the gravity value of a station. In order to determine a value for S, it requires that at least one station's gravity value in the network be known. This implies that no matter how many equations similar to equation (2.2) are formed, the

system of equations will have a rank deficiency of at least one as the result of the unknown value of offset S. The exact rank deficiency of the system will depend on the functional relationship, f(O), used. The rank deficiency of the system determines the minimum number of stations whose value of gravity is required to be known.

2.4 Internal Components of the "G" Gravity Meter

The LaCoste & Romberg "G" gravity meter can be thought of as consisting of two major components, the meter case and the meter box. The meter case which houses the meter box consists of the external casing including top, insulating material, electrical components, levelling screws, temperature probe and the heater box. The levelling bubbles, however, are part of the meter box and not part of the meter case.

The meter box is a mechanical-optical device which can be thought of consisting of four major inter-connected assemblies: gear train, measuring screw, lever linkage and optical system.

2.4.1 Meter Case

The meter case houses the meter box. The meter case is insulated to protect the meter from changes in the ambient temperature. Changes in the operating temperature of the meter box have a marked effect on the gravity meter readings [Kiviniemi, 1974]. To insure a constant operating temperature, the meter box is installed in a heater box which requires a small amount of electrical power to keep the instrument at

operating temperature. The optimum operating temperature to be ntained is determined from test procedures after which the heater is adjusted to maintain that operating temperature for the trument. A temperature probe is installed in the meter case to mit the user to verify that the instrument is at its operating perature and that the heater box is working properly. The ulation also acts as a shock absorbing material in case the gravity er were to be accidentally jarred or dropped.

The gravity meter can be used to make consistent observations only er the instrument's operating temperature has been attained and ntained for a length of time. When this occurs the instrument is d to be on-heat. The recommended length of time of being on-heat ore observations should be made is about four hours [Perry, 1980, vate communication]. If the power is interrupted for any length of e and the temperature of the instrument falls below its operating perature, the instrument is said to be off-heat. If an instrument s off-heat, it must be put back on-heat before it can be used to e additional observations. Any gravity meter differences determined the meter was off-heat must not be considered as part of rvation set. This means that the gravity meter must be on-heat ng its transportation between stations when observations are being .

he top of the meter case is removable to permit the installation of meter box. In addition, on the top will be a name plate which tifies the instrument and, generally, below the thermometer opening

will be the value of the current null or reading line for the instrument.

The meter case contains the levelling screw by which the gravity meter is levelled. For a period of time, the LaCoste & Romberg "G" gravity meters were manufactured with the thumb screws used for levelling the gravity meter located on the bottom of the meter case. But now, the instrument is being manufactured so the levelling can be adjusted via knobs that extend above the top of the meter case. This modificiation does not affect the behavior of the instrument but makes the levelling of the instrument more convenient and reduces the possibility of jarring or moving the instrument when it is being levelled [Perry, 1980, private communication].

In addition, all the electrical connections for the instrument are housed in the meter case. These include the connections for the power supply to operate the heater box and lamps and any optional electronic devices such as the electronic readout. The electronic readout is really part of the meter box since it basically consists of a set of capacitor plates installed above and below the beam and a nulling meter. As the beam moves between these plates, the change in the capacitance is recorded on the nulling meter installed in the meter case top. This nulling meter is then adjusted so when the instrument is in the null position, the nulling meter will be in its center position [Hemingson, 1980, private communication].

2.4.2 Meter Box Gear Train Assembly

The gear train assembly shown in Figure 1 consists of the dial, dial shaft, counter and the gear box. The dial, which is turned to null the instrument, has 100 equal divisions marked on it with each division corresponding to 0.01 counter units. The counter is attached to the dial shaft and is used to keep track of the number of rotations of the dial shaft with one counter unit equivalent to one complete rotation of the dial shaft. The counter records in 0.1 counter units from 0.0 to 6999.9. Physical stops within the counter prevent the dial from being rotated outside this range. On the end of the dial shaft opposite the dial is a toothed gear which drives the gears in the gear box.

There are two different types of gear boxes that have been installed in the "G" gravity meters. The original gear box installed in instruments prior to 'G-458' is referred to as the old gear box. The old gear box used a floating pivot gear system. In this system, the dial shaft with its 17 tooth gear drove a 134 tooth floating pivot gear. The floating gear, in turn, had a 20 tooth smaller gear which drove a 180 tooth gear on a shaft to which the measuring screw was attached as shown in Figure 2. The floating pivot gear was held in contact with the dial gear and the measuring screw gear by spring tension.

The gear box installed in meters built since meter 'G-458' uses a fixed pivot gear system and is referred to as the new gear box. The new gear box uses a 30 tooth gear at the end of the dial shaft which drives a 220 tooth fixed pivot gear. The fixed pivot gear in turn has

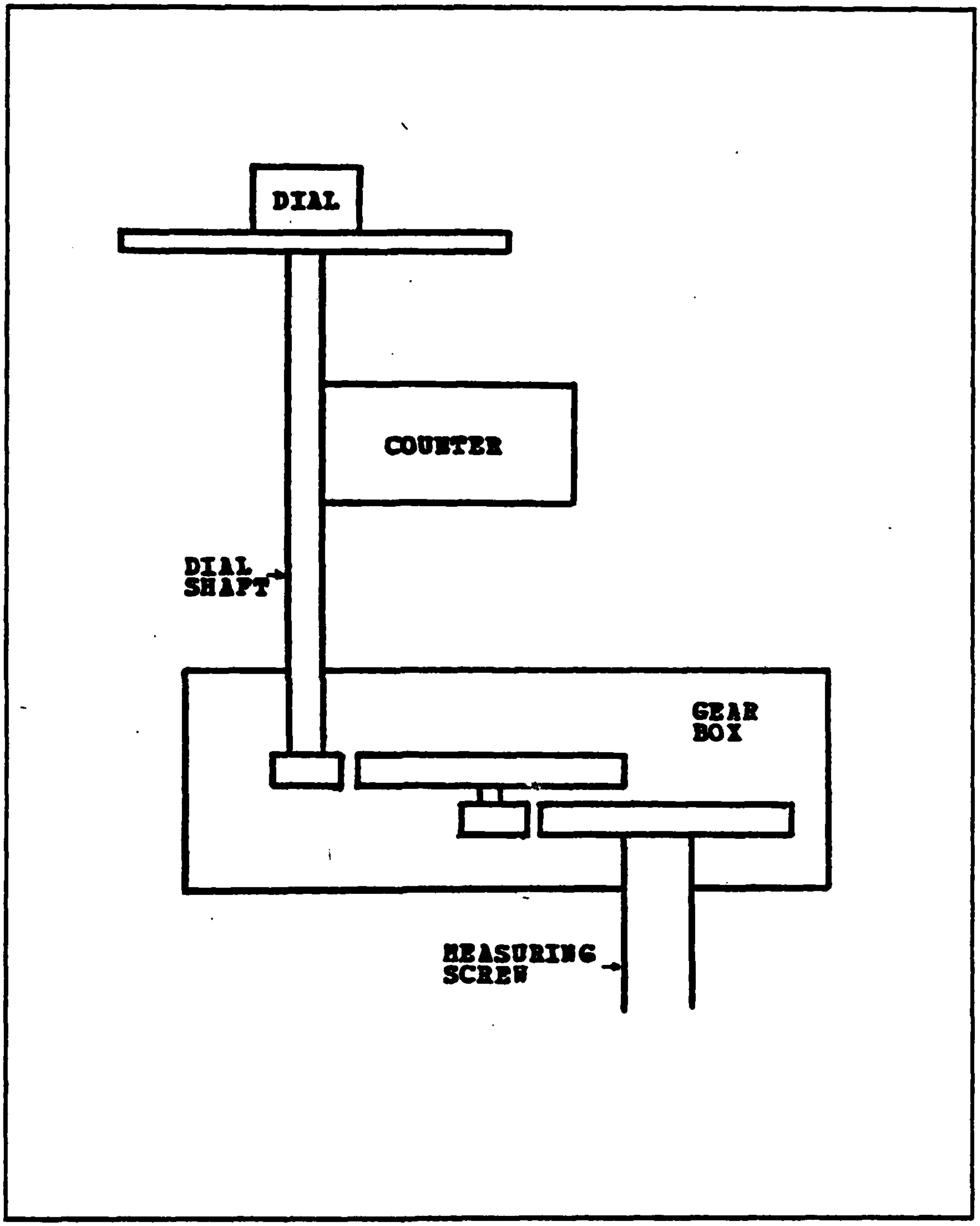


Figure 1 — Schematic illustration of the gear train assembly used in the LaCoste & Romberg "G" gravity meter.

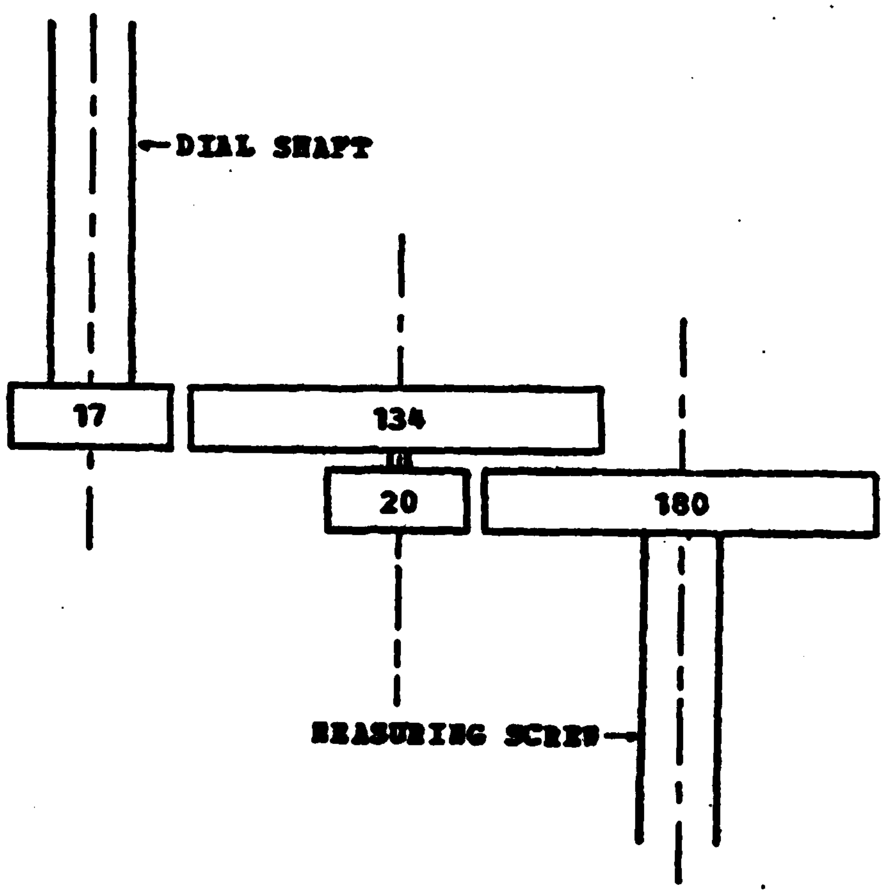


Figure 2 — Schematic illustration of the old gear box used in the LaCoste & Romberg "G" gravity meter.

a 30 tooth gear which drives a 300 tooth gear on the shaft to which the measuring screw is attached as shown in Figure 3.

With the old gear box, 1206 rotations of the dial are required to make the measuring screw rotate 17 times, a ratio of approximately 70.941:1; with the new gear box, 220 rotations of the dial results in 3 rotations of the measuring screw, a ratio of approximately 73.333:1.

Just because a gravity meter originally had an old gear box installed in it does not imply that it will always have an old gear box. If the gravity meter is returned to the factory for repairs, and the gear box needs to be replaced, a new gear box might be used as a replacement. Therefore, it is very important to know what components are presently installed in the gravity meters being used because they effect the modelling of the gravity meter's behavior. If there is any doubt, the manufacture's log on the construction of each gravity meter, which is maintained by LaCoste & Romberg, Inc., should be consulted.

2.4.3 Meter Box Measuring Screw Assembly

The measuring screw moves within a hollow shaft which has threaded fingers at the end furthest from the gear box as shown in Figure 4. The rotation of the dial causes an angular motion of the measuring screw. This angular motion of the measuring screw is converted to a linear motion by the measuring screw threaded being in contact with the stationary threaded fingers. The maximum linear motion of the measuring screw in a "G" gravity meter is on the order of 20 mm and this motion is accomplished in less than 100 turns of the measuring

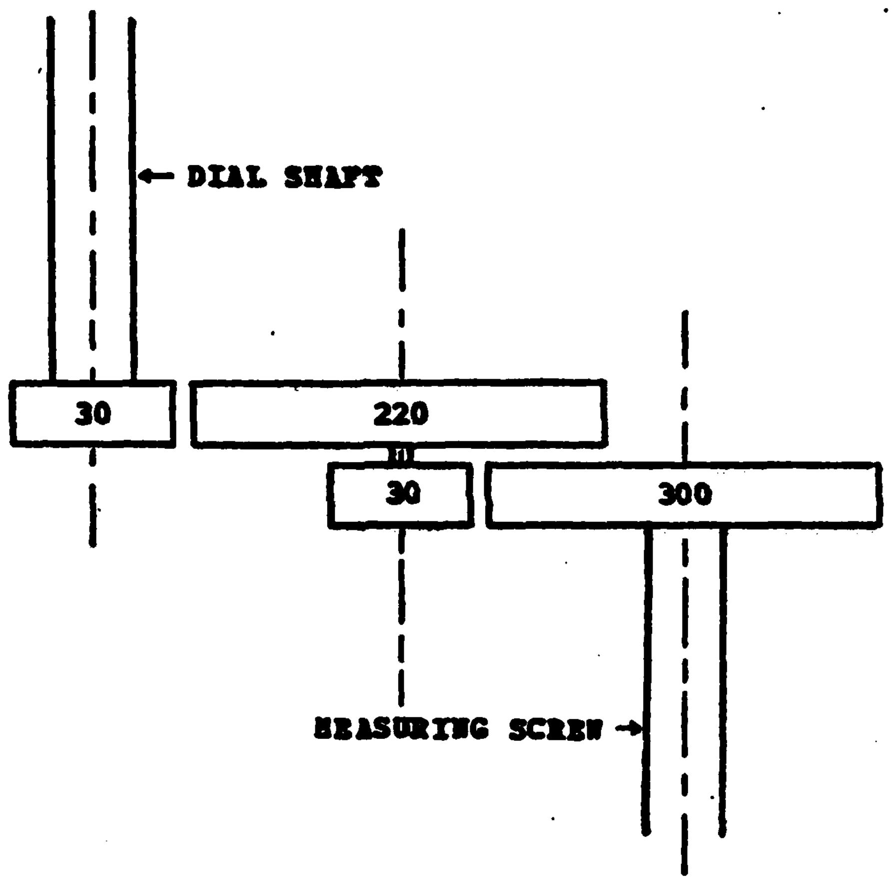


Figure 3 — Schematic illustration of the new gear box used in the LaCoste & Romberg "G" gravity meter.

screw [Perry, 1980, private communication].

At the end of the measuring screw furthest from the gear box, a hardened metal jewel in the shape of a donut is set by a press fit into the measuring screw as shown in Figure 5. The diameter of the hole in the jewel is less than 1 mm [Perry, 1980, private communication]. There are four set screws in the measuring screw which can be used for minor centering adjustments of the jewel. Since there is a spherical metal ball on the end of the lever linkage which makes contact with the jewel, it is important that the jewel is shaped and positioned such that the spherical metal ball is always in contact with the edge of the jewel's hole. If this is not the case, then the uniform rotation motion of the screw could be translated into a non-uniform motion of the lever linkage resulting in the difference between counter readings for a given gravity difference not being constant. The actual difference in counter readings would then depend on the starting position of the dial for each counter reading.

2.4.4 Meter Box Lever Linkage Assembly

The lever linkage consists of a lower lever, connecting linkage, upper lever, zero length spring, beam and beam weight as shown in Figure 6. This system is the heart of the gravity meter. Many individual parts must be assembled to create this delicate system. The connecting linkage, for example, consists of a number of flat springs screw clamped together and to other lever arms.

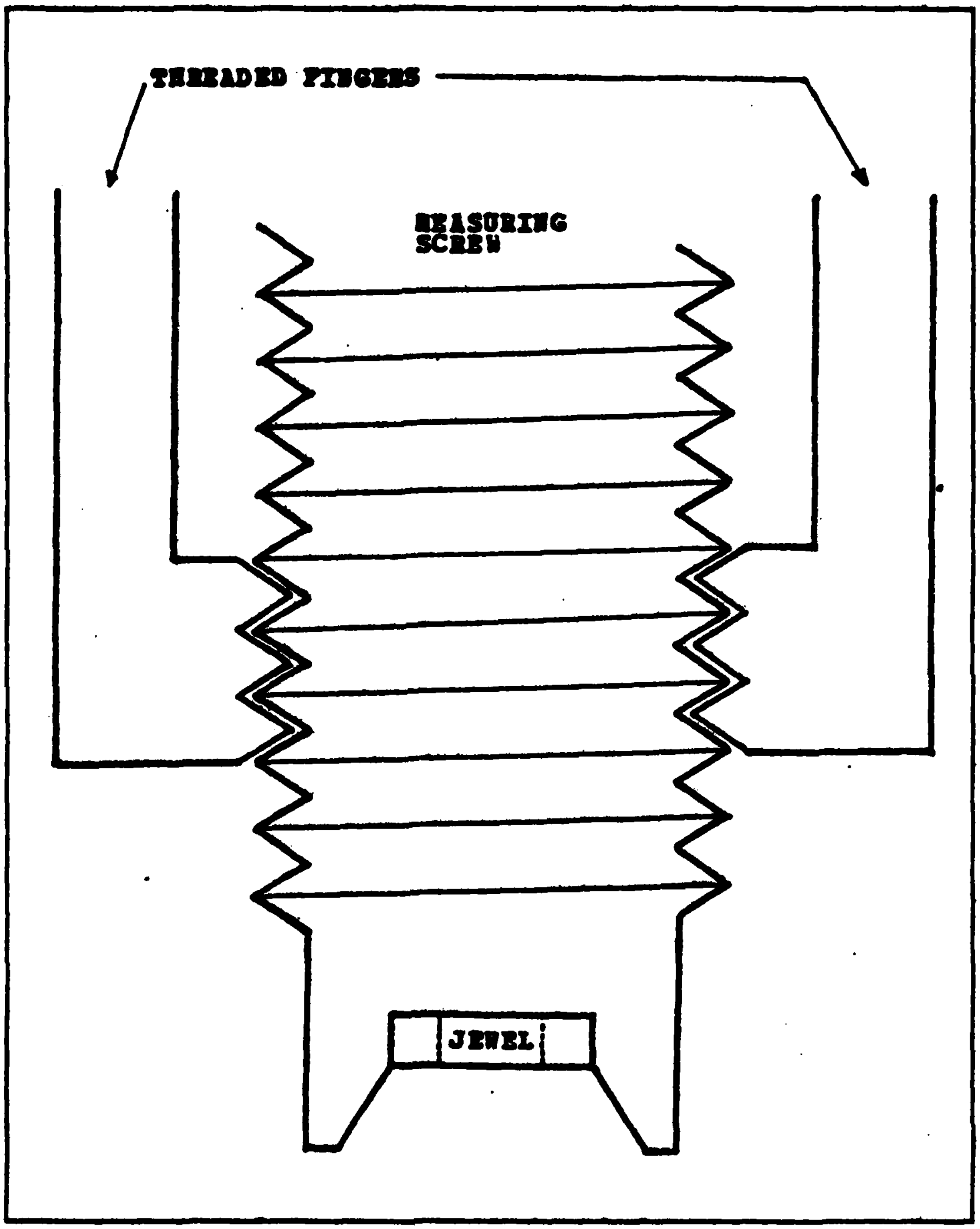


Figure 4 — Schematic illustration of the measuring screw used in the LaCoste & Romberg "G" gravity meter.

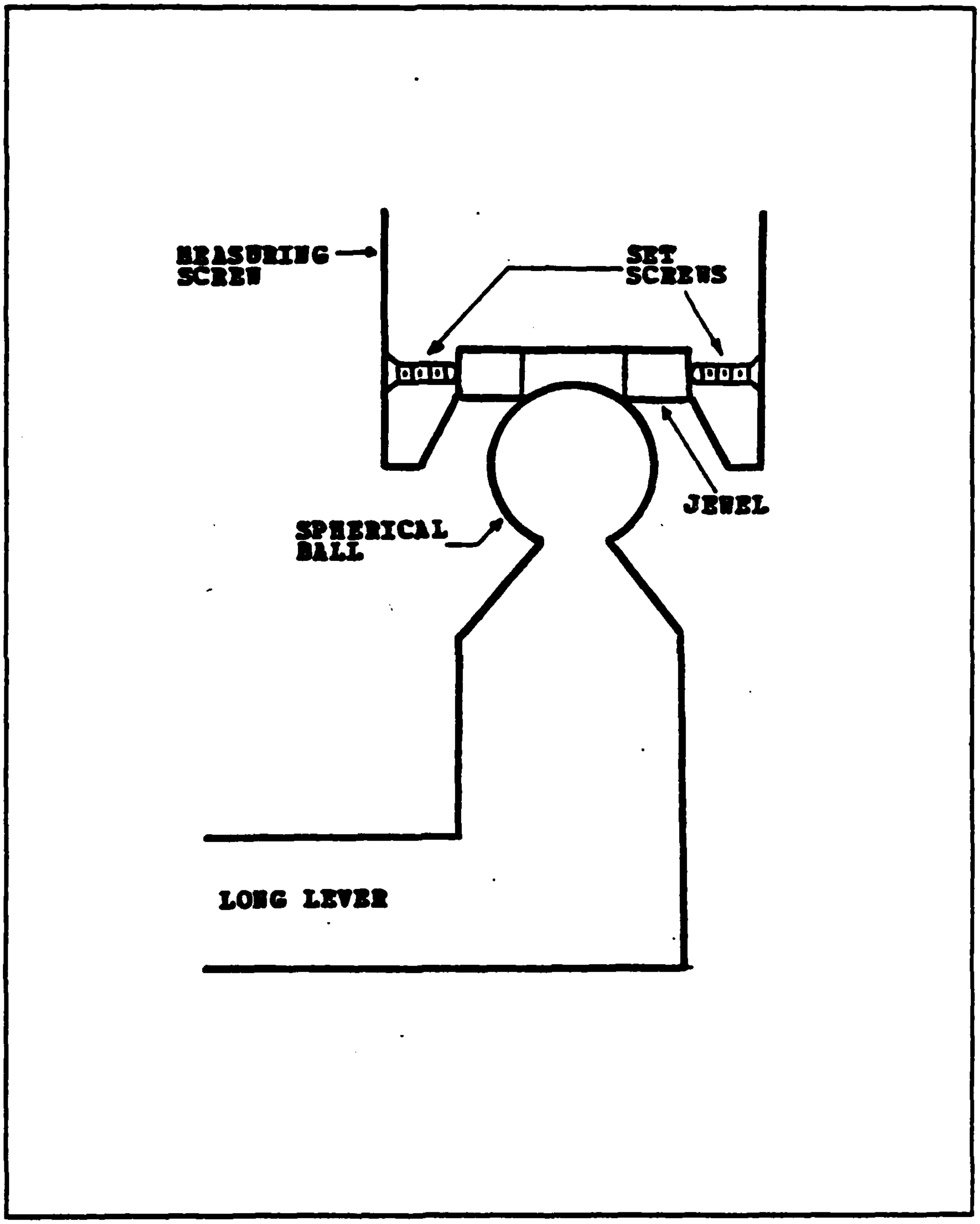


Figure 5 — Schematic illustration of connection between the measuring screw and the lever linkage used in the LaCoste & Romberg "G" gravity meter.

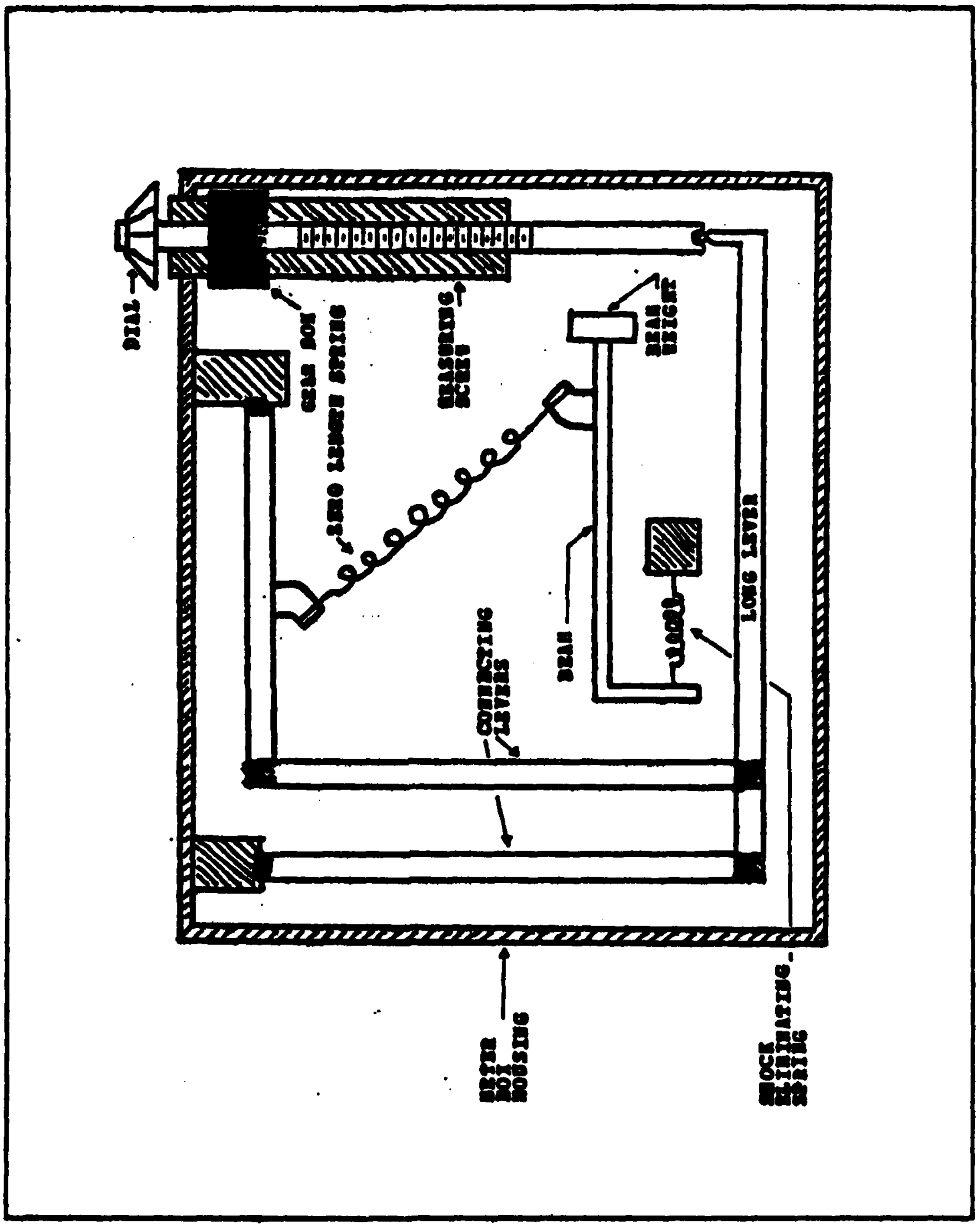


Figure 6 — Schematic illustration of the lever linkage assembly used in the LaCoste & Romberg "G" gravity meter.

The measuring screw's jewel is designed to be in continuous contact with the spherical metal ball attached to the lower lever. The spherical metal ball has been known to break loose from its support post in which case the rough edges of the support post were in contact with the jewel. This causes the difference in readings between gravity stations to act very erratic which would be a definite indication that the gravity meter needed to be repaired.

The beam with the beam weight at one end is permitted to move only a few thousands of an inch in the lateral and horizontal direction before it encounters physical stops [Hemingson, 1980, private communication]. There is an arrestment knob which permits the beam to be clamped against the stops so that damage to the system can be minimized during the transportation of the instrument.

The beam weight in addition to providing necessary mass and balance for the beam, provides the means of calibrating the instrument. How this is accomplished is explained in section 2.5.

2.4.5 Meter Box Optical System Assembly

The link between the lever linkage and the optical system is by means of what is called the ladder, which is suspended from the bottom of the beam near the beam weight. This ladder consists of two posts with a number of thin wire steps strung between the posts as shown in Figure 7.

A set of prisms direct light which has passed through the ladder onto an etched scale mounted on the meter box. The eyepiece is focused

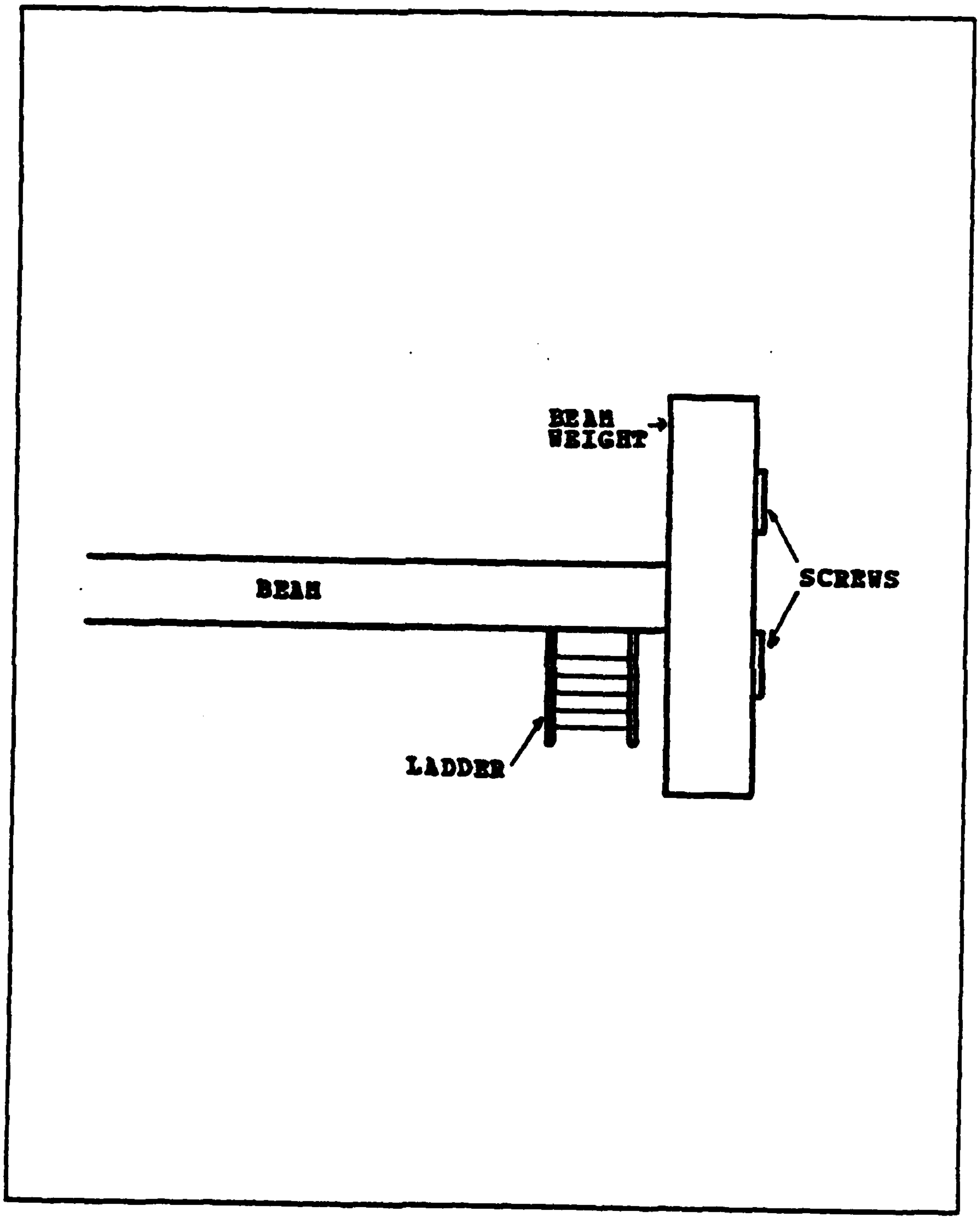


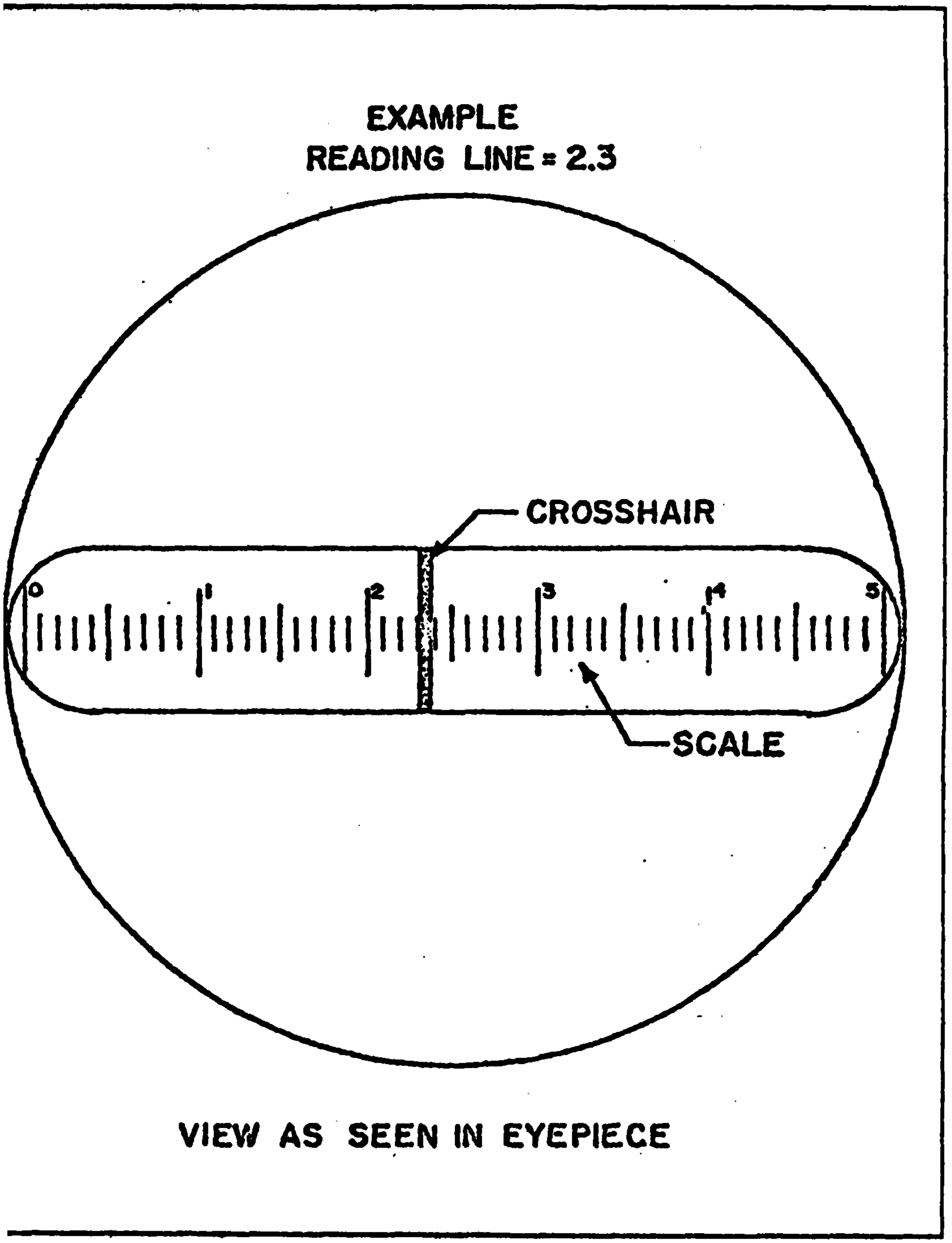
Figure 7 — Schematic illustration of the ladder assembly used in the LaCoste & Romberg "G" gravity meter.

on this etched scale which produces an image similar to the one shown in Figure 8.

The cross-hair viewed through the eyepiece is actually the shadow of one of the thin wire steps of the ladder. Adjustments are made so only one step can be viewed through the eyepiece [Hemingson, 1980, private communication]. The step that is actually viewed depends on how the prisms and light source are installed and the ladder is constructed. To insure that a step can always be viewed, the ladder is constructed with many steps.

The physical stops are adjusted so the cross-hair will move approximately six to seven scale divisions either side of the reading or null line. The reading line for the instrument is determined during the construction of the instrument following a procedure which allows the builder to deduce when the beam is in the horizontal or null position. When the actual null position does not correspond exactly with an etched scale division, the nearest etched scale division is selected as the reading line.

Two bubble levels mounted perpendicular to each other are installed on the meter box. The levels used are generally 60 second levels but the customer can request 30 second levels be installed [Hemingson, 1980, private communication]. These levels are adjusted so when they are centered, the beam is horizontally positioned between its physical stops when it is in its null position.



re 8 — Schematic view through the eyepiece of the LaCoste & Romberg "G" gravity meter.

2.5 Calibration of the "G" Gravity Meter

During the construction of the gravity meter. tests are made on various systems and assemblies in an attempt to ensure some type of uniformity in the operational behavior of the instrument after it is completed. Since minor differences in the parts used and the assembly procedure will always exist, the behavior of each instrument will be unique. However, there does exist a general characteristic behavior of the gravity meter caused by the non-linearity of the lever linkage system [Harrison and LaCoste, 1978] which can be identified. The general characteristic behavior that is sought is how differences in counter unit readings at two sites are related to the gravity difference between the two sites. The method that enables this relationship to be deduced is commonly referred to as the calibration procedure.

The calibration procedure is a two step process. The first step, which will be referred to as the factory calibration procedure, determines the general behavior of the gravity meter over its operating range by determining what will be called relative scale factors. The second step, which will be referred to as the field calibration procedure, relates the gravity difference between two stations to the difference in counter unit readings between the stations enabling what will be called the absolute scale factors for the gravity meter to be determined. The end product of the calibration procedure is the Calibration Table 1 which relates the gravity meter's counter readings to their relative values in milligals.

To determine the general characteristic behavior of the gravity meter requires that observations be made over the entire range of the counter, from 0 to 6999 counter units. One way to accomplish this would be to have the gravity meter make observations at various stations with known gravity values distributed over the entire range of the gravity meter. This of course would be time consuming, expensive and impractical. Another way to achieve the same results would be to somehow fool the gravity meter into thinking that the gravity value at a site had changed.

Since the gravity meter works on the principle that the force on a spring and the resulting change in the spring's length is related to the mass it is supporting and the acceleration of gravity on that mass, one way of achieving a change in the force without changing the acceleration due to gravity would be to vary the mass being supported. So, if there were a way to change the mass of the gravity meter's beam, then readings over the entire range of the gravity meter could be obtained. Due to the construction of the gravity meter, the gravity meter's beam acts as a lever. The effect of changing the mass of the gravity meter's beam can be achieved by changing the center of mass of the beam without actually changing the mass of the beam. An apparatus was developed which enables both the center of mass of the beam to change and masses to be added and removed from the beam. The name given to this apparatus is Cloudcroft, Jr.

2.5.1 Cloudcroft, Jr. Apparatus

Cloudcroft, Jr. is a device that consists of two shafts that can be screwed into the beam weight and a housing that is attached to the meter box. See Figure 9 for a schematic view of the Cloudcroft, Jr. apparatus. One of the shafts is threaded. On this threaded shaft is placed the range adjustment nut which can be positioned anywhere along the shaft by simply rotating that nut. The range adjustment nut acts as a counter—balance and by moving it either towards or away from the beam weight, it can be used to change the center of mass of the beam. The other shaft has an opening at one end into which a pin can be inserted. On the pertruding end of the pin is attached with a drop of glue a thin wire which supports a mass which is referred to as a bob. Two thin metal wafers called weights rest on the top of the bob.

The housing that is attached to the meter box is referred to as the bucket. It enables the weights to be lifted off and returned to the bob by a process of raising and lowering the bucket. The bucket is positioned so that when the bucket is raised, the bob will descend freely into the bucket. As the bucket is raised, each weight comes to rest on a separate ledge of the bucket, which results in the mass being removed from the beam. With this configuration, either both weights are resting on the bob, the smaller weight is resting on the bob or neither weight is resting on the bob.

With the weights being either on or off the bob, the terms of weight-on and weight-off are used respectively.

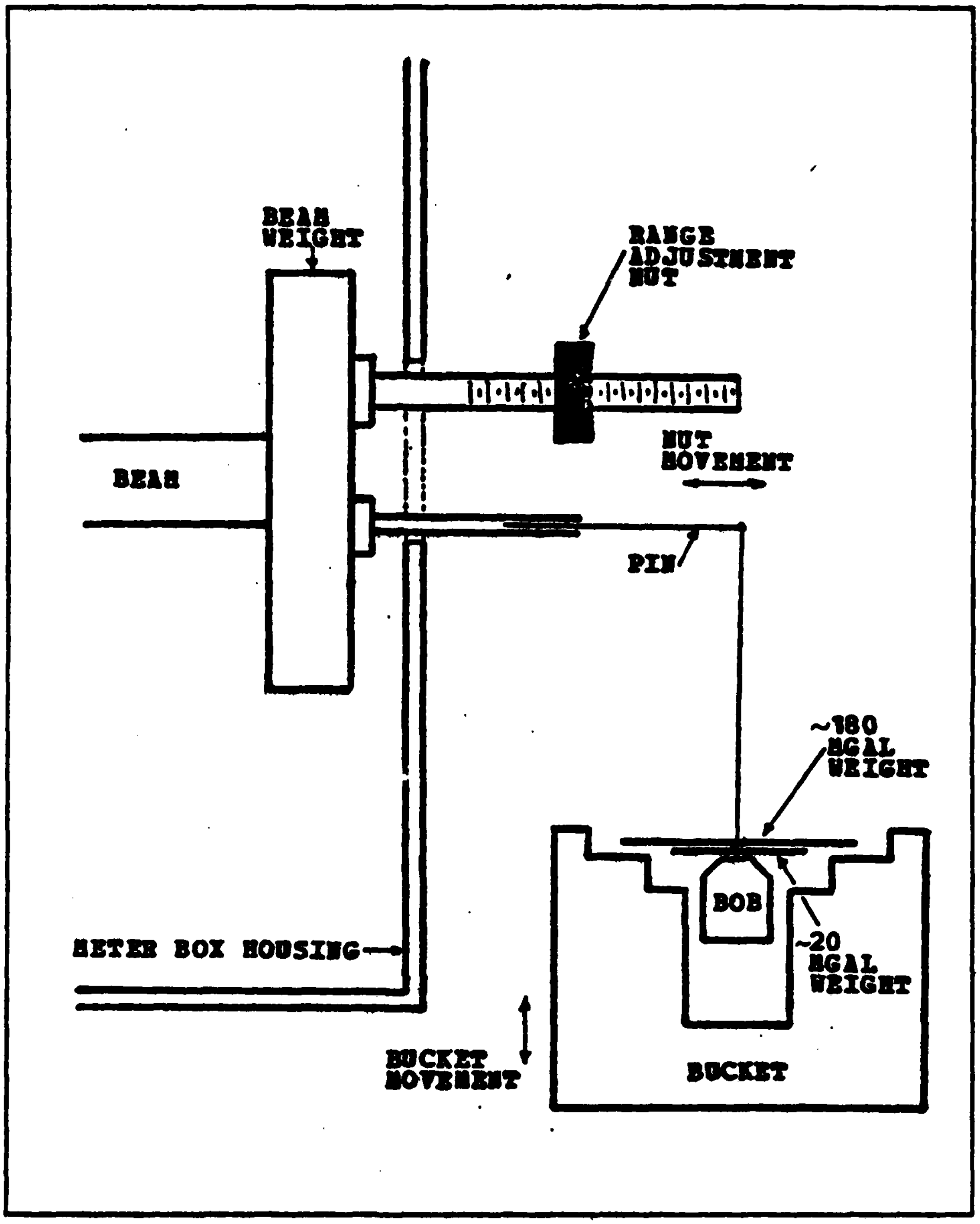


Figure 9 — Schematic illustration of the Cloudcroft Jr. calibration apparatus used in the calibration LaCoste & Romberg "G" gravity meter.

The weights used are very thin with very little mass. See Figure 10 for the approximate configuration of each weight. The weights are constructed in such way that when the larger weight is added to the beam it causes a change of approximately 180 counter units. When the smaller weight is added, it causes a change of approximately 20 counter units. Since a change of one counter unit is approximately equivalent to a change of one mgal, the larger weight is referred to as the 180 mgal weight and the smaller weight is referred to as the 20 mgal weight.

With this system, changes of approximately 20, 180 and 200 counter units can be induced in the gravity meter. And by moving the range adjustment nut along the threaded shaft by means of a fork shaped device, the gravity meter's counter can be made to read any value within its range. The Couldcroft, Jr. apparatus is installed to perform the factory calibration and then removed prior to the field calibration.

2.5.2 Factory Calibration Procedure

The factory calibration procedure is quite simple to perform but somewhat time consuming. The procedure normally takes from 10 to 12 hours to obtain the approximate 120 observations required for determining the relative scale factors over the operating range of the gravity meter. The steps involved in this procedure are:

1) the Couldcroft, Jr. apparatus is installed and the meter is put on—heat.

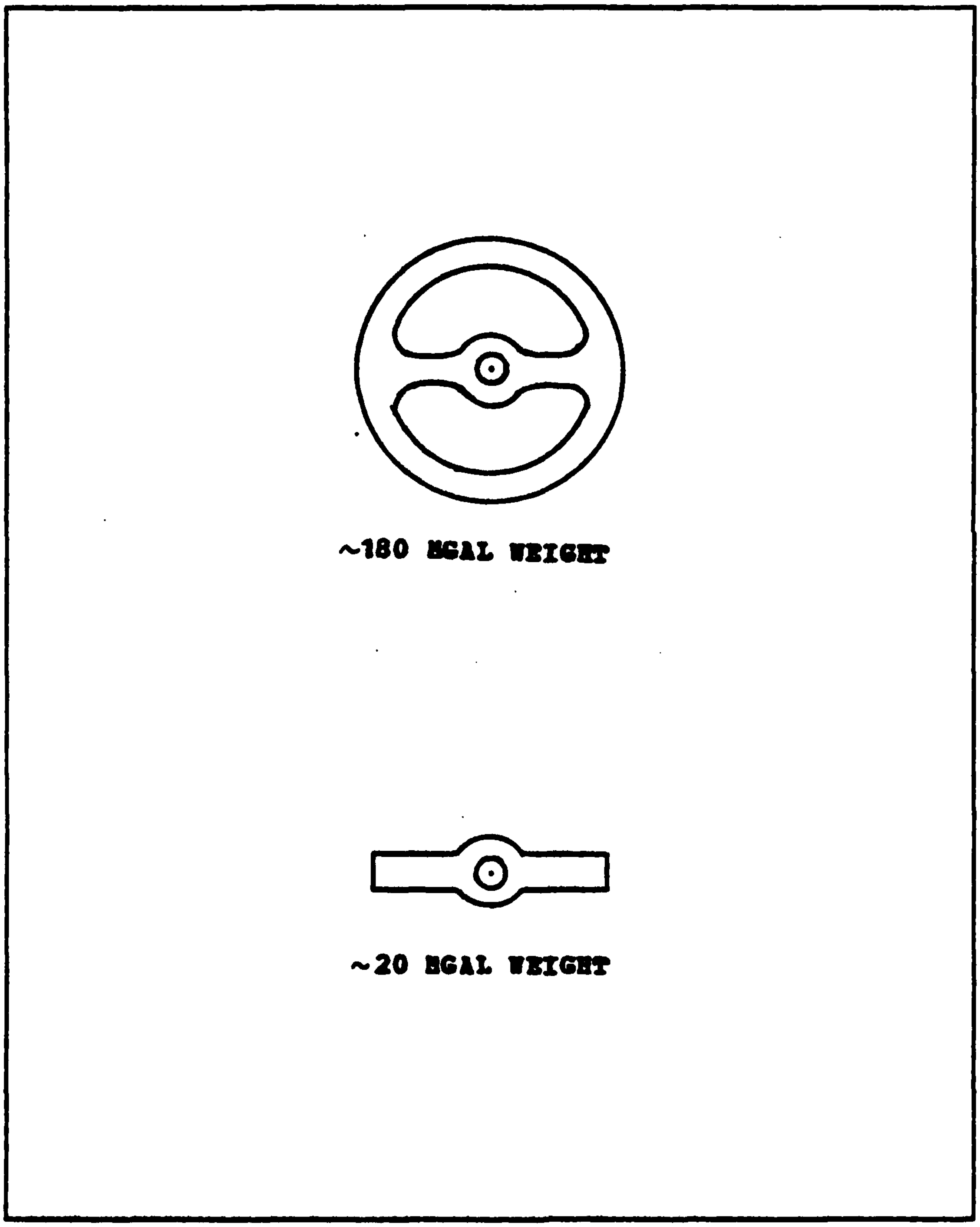


Figure 10 — Schematic illustration of the configuration of the weights used by the Cloudcroft Jr. apparatus.

2) the range adjustment nut is adjusted in the weight-off position to read a value near one end of the counter range.

3) null the instrument with the weight-off and record the reading.

4) null the instrument with the weight-on and record the reading.

5) repeat step 3).

6) repeat step 4).

7) adjust the range adjustment nut so the weight-off reading is approximately 200 dial units larger or smaller depending on which end of the counter range first set of observations were made.

8) repeat steps 3) through 7) until the opposite end of the counter range is reached.

From the recorded observations a set of differences, with each difference being the average difference between the weight-on and weight-off observations for a position of the range adjustment nut, is determined. This set of differences is then divided by an arbitrary value, generally around 200, to produce a set of relative scale factors. These relative scale factors relate how the change in gravity resulting from the addition of a constant mass varies over the range of the instrument. The set of relative scale factors obtained is assumed to be valid at the average of the weight-on and weight-off readings for each position of the range adjustment nut. The set of relative scale factors is then plotted against their average weight-on and weight-off readings. Generally, the relative scale factors over the entire range of the instrument are not permitted to vary by more than 5 parts in 1000. This is done because the graph paper used for plotting these

relative scale factors as a function of counter units at the scale desired does not permit the relative scale factor to have any larger of a variation [Perry, 1980, private communication]. An arbitrary curve that is supposed to represent the data points is drawn free hand or by using a curve template. This curve is referred to as the calibration curve. See Figure 11 for an example of the plotted data used to produce the calibration curve. The calibration curve produced will not necessarily go through all the relative scale factor data points. The discrepancy between the curve and the data points can be easily as large as 1 part in 10000 of the relative scale factor value. If the resulting calibration curve shows any unexpected strange behavior such as erratic dips or humps, attempts are made to remove the undesired behavior by changing parts of the instruments such as the gear box and/or the measuring screw [Perry, 1980, private communication]. The last resort would be to modify or rebuild the lever linkage assembly. If a component is replaced, such as a gear box, it does not mean that the one removed is bad and cannot be used again. Many times, components removed from one instrument whose the calibration curve was not satisfactory will not produce any adverse effects in the calibration curve when re-installed in another instrument [Perry, 1980, private communication].

The Calibration Table 1 for a gravity meter is determined prior to the delivery of the gravity meter to the customer and is not altered unless the gravity meter is returned to the factory and a major modification, such as, replacing the measuring screw, gear box or long

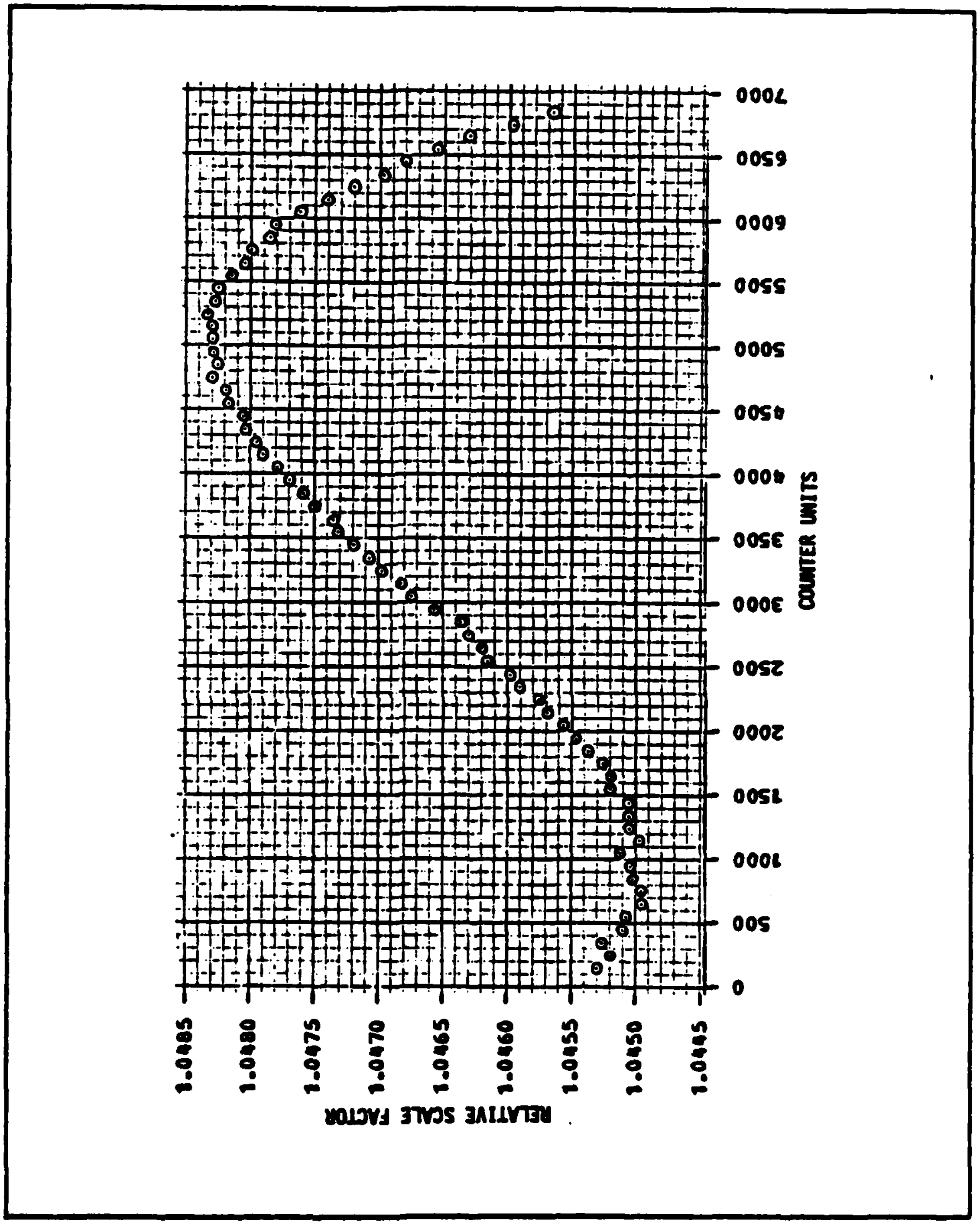


Figure 11 — A plot of relative scale factors for the LaCoste & Romberg "G" gravity meter "G-220".

lever [Perry, 1980, private communication]. This would imply that there exists only one Calibration Table 1 for each gravity meter which is valid at any time. The Calibration Table 1 is the result of factory calibration procedures. The truth of the matter is that this is not always the case which leads to a lot of confusion.

It appears that the Geodetic Survey Squadron out of F. E. Warren, AFB in Wyoming, which is responsible for making the majority of the gravity meter observations in the United States, produces their own Calibration Table 1. The difference between their Calibration Table 1 and the one provided by the manufacturer is generally just a constant scale factor applied to the values in milligal. A new Calibration Table 1 is produced periodically because it is believed that the gravity meter's calibration changes with time [Beruff, 1980, private communication].

When the Geodetic Survey Squadron concludes that the calibration of the gravity meter has changed, it determines the scale factor that it wishes to be apply to the value in milligal and often request LaCoste & Romberg to produce a new Calibration Table 1 for them using the same format as the original Calibration Table 1 [Perry, 1980, private communication]. In the process, due to round-off, the new Geodetic Survey Squadron's Calibration Table 1 values in milligal are not an exact scale factor multiple of the original Calibration Table 1 supplied with the gravity meter. This makes it very difficult to determine which Calibration Table 1 should be used because there no remark the Calibration Table 1 to indicate that the table has been

modified. This perpetuates the notion that the calibration of the gravity meters changes with time.

It is known that by making changes in the lever linkage assembly, the general characteristics of the calibration curve can be changed. The prime example of this is the gravity meter 'G-253'. This gravity meter was especially constructed so its calibration curve was flat, that is all the relative scale factors had the same value. This is probably not exactly true but to within 2 to 3 parts in 10000, the relative scale factors are the same. Given enough time and funds any "G" gravity meter could be constructed with a flat calibration curve [LaCoste, 1980, private communication].

Whether "G" gravity meters with flat calibration curves are better than those that do not have flat calibration curves is hard to say because only one "G" gravity meter is known to have such a characteristic.

Once an acceptable calibration curve has been obtained, the instrument is sent for its field calibration.

2.5.3 Field Calibration Procedure

The purpose of the field calibration procedure is to enable the absolute scale factors to be determined. The absolute scale factors relate the counter units to their values in milligal. This is accomplished by taking the instrument to an area near Cloudcroft, New Mexico where two stations exist, Cloudcroft and La Luz, which have a gravity difference of about 242 mgal. A number of repeated

observations are made between these two stations. From these observations an average counter difference is determined. The actual gravity difference between Cloudcroft and La Luz is critical in as much as the better the value, the closer the value in milligal found in the Calibration Table 1 will reflect true milligal units. This is important only if the Calibration Table 1's values in milligal are to be used without being adjusted.

Using an assumed value for the gravity difference between these two stations, a field scale factor is computed. It relates the counter unit difference to the value in milligal difference by dividing the gravity difference by the average counter difference. Although the field scale factor determined in this manner is truly only valid over the range of the readings used in its determination, the Calibration Table 1 is assumed to be valid for the entire range of the gravity meter.

2.5.4 Construction of the Calibration Table 1

After the factory and field calibration procedures have been completed, the Calibration Table 1 is produced. See Table 2 for an example of a Calibration Table 1 as supplied by LaCoste & Romberg, Inc. It is very important to understand how the Calibration Table 1 is produced and what type of information this table does and does not contain. This table relates counter readings to value in milligal. By reading relative scale factor values off of the plotted calibration curve at intervals of 100 counter units and starting at 50 counter

units, a table of counter units and relative scale factors is produced. These relative scale factors are assumed to be valid for plus/minus 50 counter units from the point on the calibration curve that the reading was made. These relative scale factors are then all scaled by the field scale factor to produce what is referred to as the factor for interval. The factors for interval are assumed to be valid for plus/minus 50 counter units. From this information, the Calibration Table 1 is produced which relates the counter readings to value in milligal via the factors in interval.

In the Calibration Table 1, the factor for interval is assumed to be valid for a range of 50 counter units either side of its corresponding counter reading. The value in milligal for a counter reading is obtained by multiplying the factor for interval by 100 counter units, which is the difference between two consecutive counter readings, and adding it to the previous value for the value in milligal. It should be noted that the value in milligal is derived from the factor for interval values and not the converse. If one assumes that the standard error of the observed difference of gravity meter readings between Couldcroft and La Luz is on the order of 0.025 counter units, then this implies that the field scale factor determined and the corresponding factor for interval of the Calibration Table 1 is accurate to about 1 part in 10000.

le 2 — Calibration Table 1 for the LaCoste & Romberg "G" gravity meter "G-220".

TABLE 1

MILLIGAL VALUES FOR LACOSTE & ROMBERG, INC. MODEL G GRAVITY METER #G- 220

COUNTER READING*	VALUE IN MILLIGALS	FACTOR FOR INTERVAL	COUNTER READING*	VALUE IN MILLIGALS	FACTOR FOR INTERVAL
000	000.00	1.06106			
100	106.11	1.06094	3600	3821.64	1.06357
200	212.20	1.06083	3700	3928.00	1.06369
300	318.28	1.06074	3800	4034.36	1.06381
400	424.36	1.06065	3900	4140.75	1.06392
500	530.42	1.06060	4000	4247.14	1.06403
600	636.48	1.06057	4100	4353.54	1.06412
700	742.54	1.06057	4200	4459.95	1.06420
800	848.60	1.06059	4300	4566.37	1.06426
900	954.66	1.06063	4400	4672.80	1.06428
1000	1060.72	1.06067	4500	4779.23	1.06430
1100	1166.79	1.06074	4600	4885.66	1.06431
1200	1272.86	1.06080	4700	4992.09	1.06432
1300	1378.94	1.06088	4800	5098.52	1.06433
1400	1485.03	1.06097	4900	5204.95	1.06433
1500	1591.12	1.06104	5000	5311.39	1.06433
1600	1697.23	1.06113	5100	5417.82	1.06431
1700	1803.34	1.06123	5200	5524.25	1.06430
1800	1909.46	1.06128	5300	5630.68	1.06427
1900	2015.59	1.06137	5400	5737.11	1.06423
2000	2121.73	1.06146	5500	5843.53	1.06418
2100	2227.89	1.06156	5600	5949.95	1.06412
2200	2334.03	1.06169	5700	6056.36	1.06403
2300	2440.20	1.06182	5800	6162.76	1.06391
2400	2546.38	1.06197	5900	6269.15	1.06376
2500	2652.53	1.06213	6000	6375.53	1.06360
2600	2758.79	1.06228	6100	6481.89	1.06343
2700	2865.02	1.06242	6200	6588.23	1.06324
2800	2971.26	1.06255	6300	6694.56	1.06304
2900	3077.52	1.06268	6400	6800.86	1.06294
3000	3183.79	1.06278	6500	6907.14	1.06261
3100	3290.06	1.06289	6600	7013.41	1.06239
3200	3396.35	1.06301	6700	7119.64	1.06212
3300	3502.65	1.06314	6800	7225.86	1.06181
3400	3608.97	1.06328	6900	7332.04	1.06146
3500	3715.30	1.06343	7000	7438.18	

* Note: Right-hand wheel on counter indicates approximately 0.1 milligal.

10-11-79
RP DELTA=[illegible]x243.9mgls.

2.6 Instrumental Error Source

Whenever the "G" gravity meters behavior differs from that predicted by a linear interpolation within the Calibration Table 1, there are two possible explanations. One explanation is that the anomalistic behavior is in fact present but the Calibration Table 1 does not contain this information. In this context, short wave length refers to wave length less than approximately 200 counter units. This situation occurs when the short wave length behavior cannot be represented by the long wave length information present in the Calibration Table 1. This type of systematic error could be accounted for by additional parameters in the mathematical model. An other explanation is that the anomalistic behavior is erratic and random in nature and thus impossible to be modelled. The major error sources that fall into either of these two categories are periodic screw effect, tares and instrumental drift.

2.6.1 Periodic Screw Effect

Due to the construction of the "G" gravity meter, there is a possibility that an angular rotation of the dial will not produce a strictly linear motion of the measuring screw. The departure from the linear motion could be due to periodic errors in the measuring screw, eccentricity in the measuring screw resulting in a wobble or the non-linearity of the lever linkage assembly [Harrison and LaCoste, 1978]. If the periodic error were in the measuring screw system and could be related to the position of the dial and the counter, then

there might be a way of modelling this effect.

There are two places in the gravity meter in which this type of effect could be introduced. One place is in the gear box [Kiviniemi, 1974; Harrison and LaCoste, 1978] and the other is at the point of contact between the measuring screw and the lever linkage.

The gear box could introduce a periodic effect into the observations due to the eccentricities in the gears of the gear box [Kivineimi, 1974]. The period of this effect would depend on which gear box was installed in the gravity meter. If the gravity meter has an old gear box installed in it, then periods of 1206, 1206/17, 134/17 and 1 counter units could be present. If the gravity meter has a new gear box installed in it, then periods of 220, 220/3, 22/3 and 1 counter units could be present.

The other place that a periodic effect could be introduced is at the point of contact between the measuring screw and the lever linkage. This results when the ball on the lever linkage and/or the hole in the hardened jewel is not spherical or circular in shape. If this were the case, then each rotation on the measuring screw would produce a type of period effect. For the old gear box, this effect would occur every 1206/17 counter units, while for the new gear box, this effect would occur every 220/3 counter units. Note that the period of this period effect is a function of the which gear box is installed in the gravity meter.

2.6.2 Tare

Tare is a term which refers to unexplained changes in the reading level of the "G" gravity meter. A tare in the gravity meter is believed to be the result of small shifts of the components lever linkage that are screw clamped together [Burris, 1980, private communication]. Tares by nature are unpredictable but are easily introduced. A rapid deceleration or acceleration of the gravity meter is a common cause that will introduce a tare. This occurs when the gravity meter is dropped or jarred especially when the beam is not clamped. Therefore, it is very important that the arrestment knob be turned fully clockwise, so that the beam is clamped whenever the gravity meter is being moved.

Large tares, on the order of 150 μgal or larger, are generally easy to detect. But smaller tares can be very difficult to identify. Any gravitv meter tie suspected of containing a large tare should be removed from the observation set. But there is little that can be done for the ties that contain the undetected small tares.

2.6.3 Instrumental Drift

The drift of the "G" gravity meter is not totally understood at this time. There appears to be no mechanical reason why readings made with a properly adjusted "G" gravity meter should change with time other than as a result of tares being introduced [Perry, 1980, private communication]. It is believed that the so called instrumental drift is the cumulative result of a number of small tares in the gravity

meter [Uotila, 1974; Burris, 1980, private communication]. The tares occur randomly rather than uniformly which makes modelling of such an effect very difficult, if not impossible.

CHAPTER THREE

GRAVITY METER OBSERVATIONS

3.1 Observations Used

For this study, observations were obtained from two governmental organizations: the National Geodetic Survey of the National Ocean Survey of the National Oceanic and Atmospheric Administration of the United States Department of Commerce and the Geodetic Survey Squadron stationed at F.E. Warren AFB, Wyoming of the Defense Mapping Agency Hydrographic/Topographic Center. The data obtained consist of over 4500 gravity meter observations made with 25 different 'G' gravity meters and 2 different 'D' gravity meters. The majority of the observations were made along the United States Mid-Continent Calibration Line which runs along the Eastern side of the Rocky Mountains with stations in Texas, New Mexico, Colorado, Wyoming, Montana and North Dakota. All observations were made during the period 1975-1980 and were received in the form of copies of the original observation sheets. See Table 3 for a listing of the gravity meters used. See Figure 12 for the geographical location of the stations in the network and how they are interconnected.

The information supplied on the observation sheets consisted of the name of the observer, the instrument(s) being used, the station name,

Table 3 — Summary of the gravity meters used in study.

Gravity Meter Number	Date on Calibration Table 1	Number of Observations	Observations Made From	To
G-10	25/10/60	164	03/08/79	29/10/80
G-44	25/04/63	14	10/02/78	15/02/78
G-47	22/05/63	15	10/02/78	15/02/78
G-50	15/06/63	17	11/08/78	11/08/78
G-68	23/03/64	61	05/10/76	26/10/76
G-81	07/08/64	427	17/04/75	26/10/76
G-81a	17/10/77	171	11/01/78	30/10/80
G-103	07/09/65	125	11/01/78	02/10/79
G-111	25/03/66	427	17/04/75	26/10/76
G-113	28/03/66	17	11/08/78	11/08/78
G-115	09/05/66	369	17/04/75	10/11/75
G-115b	14/02/78	300	11/01/78	09/02/80
G-123	22/10/79	39	18/09/79	02/10/79
G-125	17/10/66	140	25/04/78	05/02/79
G-130	18/10/66	19	03/08/79	03/08/79
G-131	15/05/78	350	27/03/78	16/05/80
G-140	24/02/67	17	12/08/78	12/08/78
G-142	14/03/67	77	07/01/78	02/10/79
G-157	10/08/67	435	17/04/75	26/10/76
G-157c	25/01/78	172	11/01/78	30/10/80
G-175	30/04/68	17	12/08/78	12/08/78
G-176	19/04/68	27	09/02/78	17/02/78
G-191	27/01/69	16	07/01/78	25/04/78
G-220	11/10/78	266	13/08/78	09/02/80
G-253	09/10/78	114	27/03/78	15/11/79
G-268	15/05/78	237	27/03/78	09/02/80
G-269	29/06/71	147	09/02/78	02/10/79
D-17	*	182	19/20/77	23/06/80
D-43	*	126	12/05/80	23/06/80

All dates are given in day, month, year order.

a — Calibration Table 1 changed due to addition of electronic readout on 18 October 1977.

b — Calibration Table 1 changed due to replacement of long lever on 27 October 1977.

c — Calibration Table 1 changed due to addition of electronic readout on 30 August 1977.

* — No Calibration Table 1 is provided with 'D' gravity meters since the scale factor is assumed to be a constant [LaCoste & Romberg, 1979b].

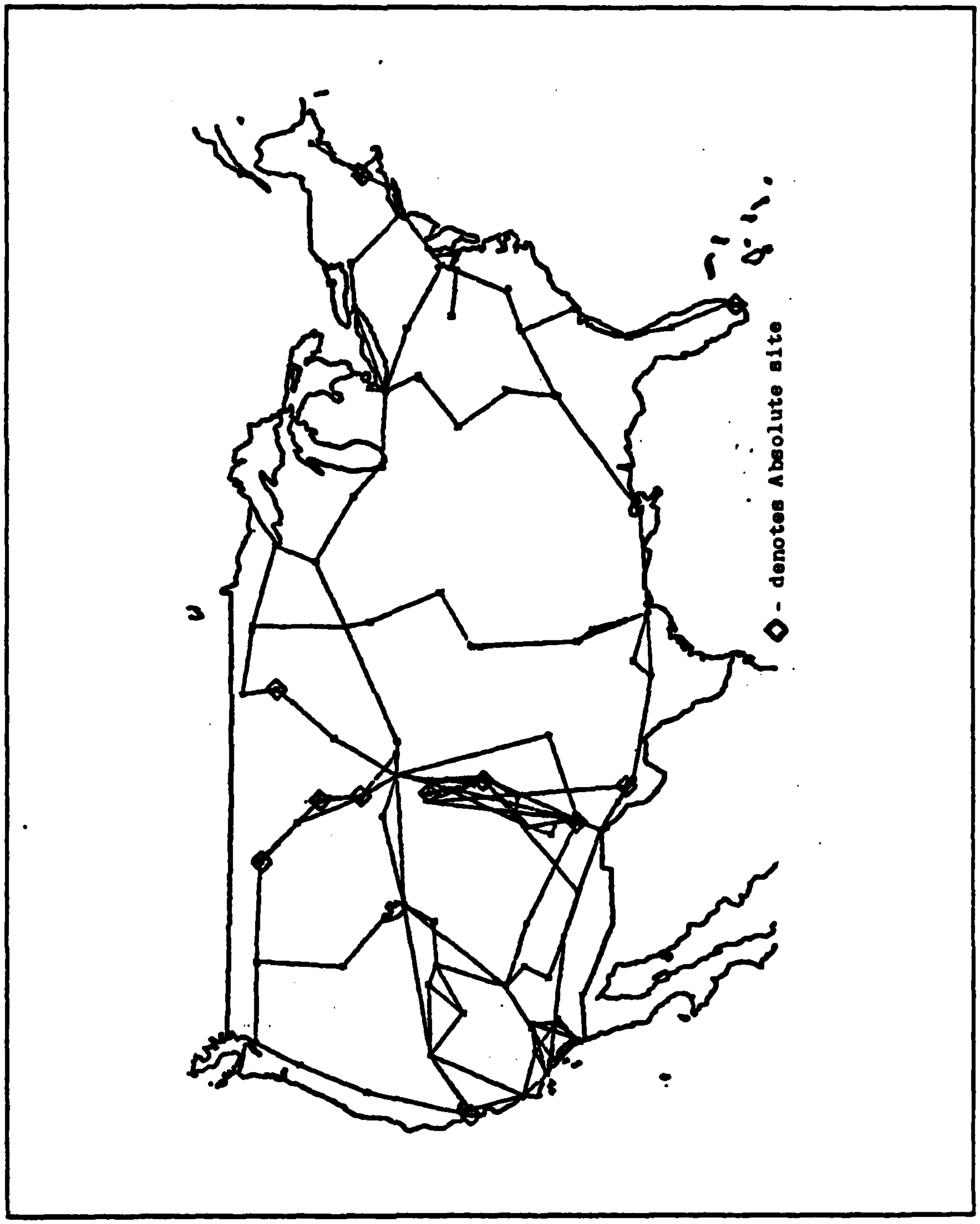


Figure 12 — Geographical distribution of gravity stations and gravity ties for the United States gravity base station network.

sometimes a station identification number, date and time of each observation to the minute, observed dial reading in dial units, height of gravity meter above or below the station, station coordinates and any remarks concerning unusual operating conditions or instrument behavior. The time was given either in Universal Coordinated Time or in local standard time with the correction needed to obtain Universal Coordinated Time. The coordinates of the station were given as latitude, longitude and elevation with the latitude and longitude generally given to the nearest 0.1 minutes and the elevation given to 0.01 meters or equivalent. See Figure 13 and Figure 14 for sample of observation sheets.

3.2 Observational Procedure

The observational procedure recommended is outlined in Land Gravity Surveys, DMAHTC/GSS-TM-9, Preliminary Edition, October 1979 on page 4-1 which basically states that

1) A valid set of observations consists of those made by one observer. This is necessary to eliminate parallax and other observer peculiarities. The gravity meter must have been at operating temperature for at least 6 hours prior to beginning observations and during the observations the operating temperature must be maintained.
2) The gravity meter may be placed directly on any smooth, hard, level surface for observing. If any of these conditions are not met, then the gravity meter should be placed on the levelling

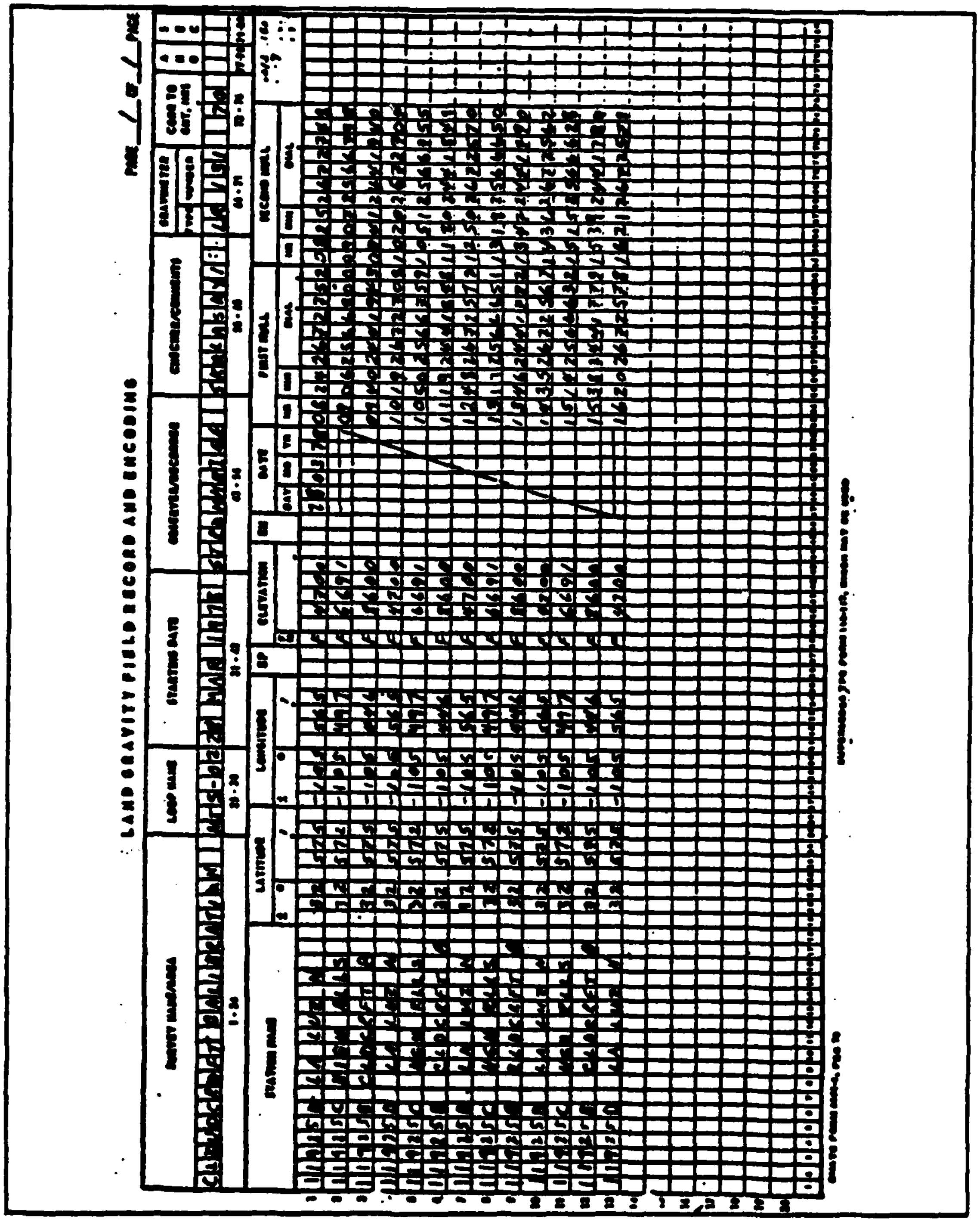


Figure 13 — Example number 1 of a gravity meter field observation sheet.

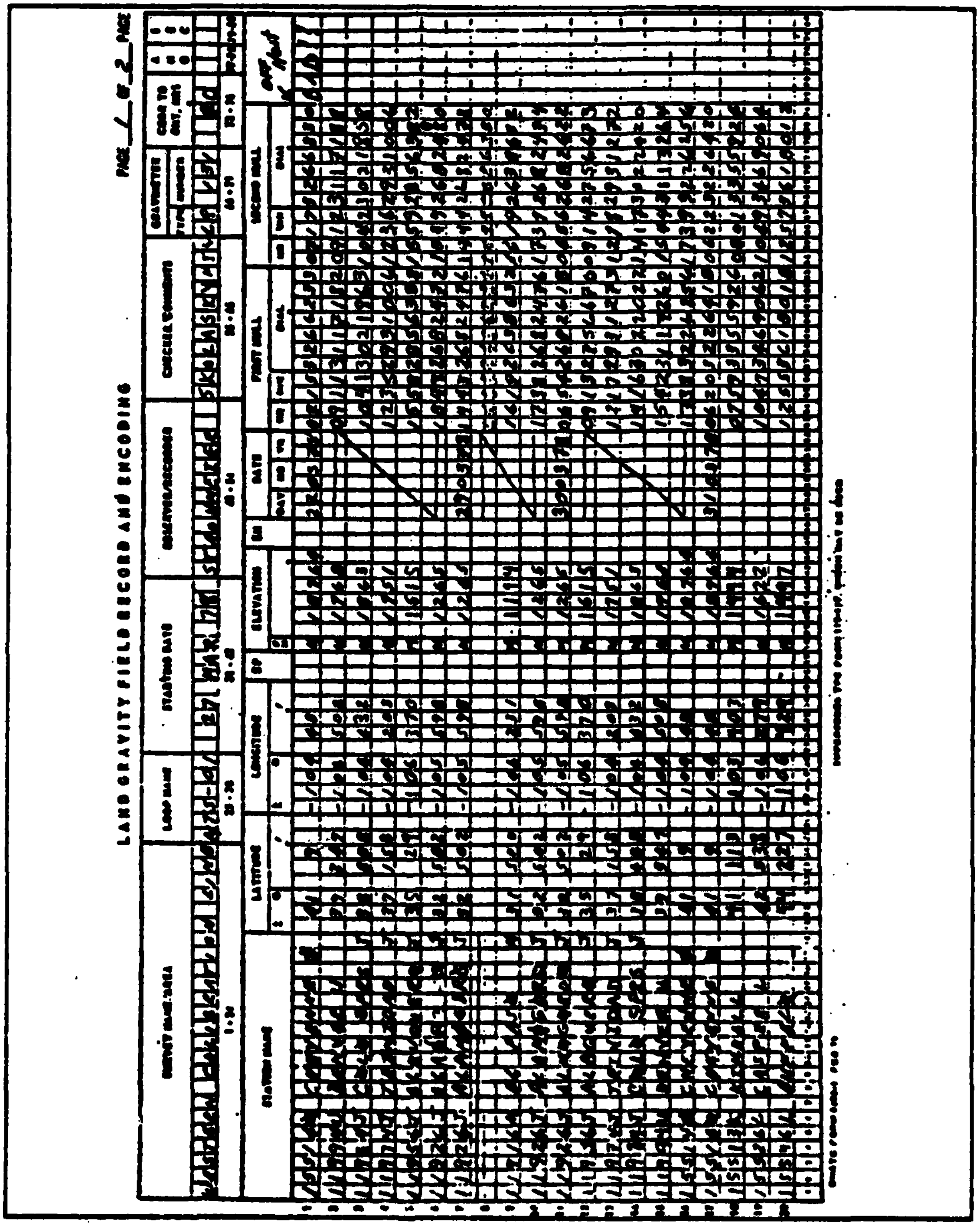


Figure 14 — Example number 2 of a gravity meter field observation sheet.

disk which must be firmly seated to eliminate any movement while the observation is being made.

3) The gravity meter must be levelled and then rough nulled. Rough nulling is accomplished by turning the arrestment knob counter-clockwise which unclamps the balance beam and then rotating the dial until the beam is off its stops but not necessarily to the reading line. The rough nulling condition should exist for approximately 5 minutes before an observation is made. During this time, the observer will enter station description information. The observer must keep the sun from shining on the gravimeter because the heat might cause distortion in the level vial assembly. When finished with the observation, the gravity meter's balance beam must be clamped by turning the arrestment knob clockwise and the gravity meter returned to its carrying case with the carrying case lid closed to prevent the gravity meter from being tipped over by the wind.

4) The gravity meter is nulled by approaching the reading (nulling) line from the down-scale (left) side to the up-scale (right) side. The null position is the coincidence of the left edge of the cross-hair with the reading line. If the observer overshoots the reading line, the dial must be offset 180 degrees down-scale and the reading line approached again. This must be done to eliminate any backlash in the dial gear system.

5) A valid observation at a station consists of two consecutive nulling, no more than 4 minutes apart, that agree to 0.01 counter

units.

Of the some 4500 gravity meter observations received, how many of these observations were obtained following the procedure outlined above is unknown. But what is known is that some of the observers did not follow that procedure. Some of the data received consisted of a single null reading per observation. This, in itself, is not necessarily bad. But, the reason for the two consecutive nullings is to provide a method of detection of blunders in nulling, reading, and/or recording. Without the second nulling, the detection of these types of blunders becomes impossible. In addition, no information is available concerning the repeatability of observations made with the instrument.

Another practice, known to occur but not how often, is that of not actually performing the second nulling [Beruff, 1981, private communication]. Instead, a type of quasi-nulling is performed. After the first nulling has been performed, the second nulling consisting of the dial being backed off at least the required 180 degrees and then the dial being set back at the first nulling position. The cross-hair is checked and if acceptable, the second nulling recorded is made identical to the first reading. What information this type of procedure provides, if any, is not clear. But this type of practice is not recommended and should not take place.

Another practice which is not uncommon is the inconsistent recording of the height of instrument [Wessells, 1980, private communication]. This occurs when for some visits to a station, the levelling disk is used, while for other visits to the same station, the levelling disk is

not used and no height of instrument is recorded. The levelling disk can elevate the gravity meter as much as 5 cm which, if the gradient of gravity is assumed to be that of normal gravity, would result is a systematic gravity change of about 15 μgal. For this reason, the levelling disk should always be used when making gravity meter observations or the height of instrument of the gravity meter should be properly recorded.

3.3 Working with Gravity Meter Data

All gravity meter data used was received as copies of the original field observation sheets. The vital information necessary to perform the gravity base station network adjustment was extracted and encoded for use in the computer.

The information encoded was the station information, observation information, and instrument information. The station information included the station's identification code, the station's name, and its location given by its latitude and longitude to 0.1 minutes and its elevation in meters. The observation information included the recorded time of the each nulling to the minute and its corresponding observed counter reading. The time recorded was either in Universal Coordinated Time (UTC) or in local standard time with the correction needed to obtain UTC. The instrument information consisted of the identification number of the gravity meter used in making the observations along with an arbitrary data set number which was assigned to each set of observation sheets as they were received.

The encoded data were then visually checked for agreement with the
iginal data and any errors corrected. Each observation, which
ierally consisted of two separate nullings, is the average of their
ll readings. Ideally, each nulling is made independently. The
rpose of the two independent nullings is to check for blunders in the
lling process and in the recording of the reading. The time of each
servation is the average of the time for the nullings. The resulting
lues were used as the observation and the time of the observation in
e adjustment program. In the process, the time which was originally
ven in year, month, day, hour and minute format was converted into a
re convenient form, its Julian Date.

The way the time of the nullings is actually recorded should be made
iformly. Either all times are given in local standard time with the
rrection needed to obtain UTC, or they are given in UTC. The problem
th recording the time of the nulling in local standard time is that
e UTC correction varies with the location of the station and the time
the year that the nulling is made in the United States. It is
commended that each gravity meter have a small electronic 24 hour
gital display clock which also displays the current date affixed to
which would be set to UTC. Then the time of the observation could
sily be recorded in UTC without worrying about time zone or seasonal
anges in the local standard time.

But by far the most confusing, yet very important, information is
station information itself. Generally, complete station
formation is not provided on the field observation sheets. Instead,

just enough station information is provided to identify the station so its gravity station description or site description can be located. On the gravity station description form is the detailed information about the exact location of the gravity station which gives its latitude, longitude and elevation with a word description of its location plus a diagram/photograph of the station's location. See Figure 15 and Figure 16 for examples of gravity station description forms.

The confusion develops when the information on the field observation sheet is not sufficient to locate its gravity station description, if it exists, or when the information of the field observation sheets does not completely agree with the gravity station description information. The latter problem occurs most often when the latitude, longitude and elevation information is not given on the field observation sheet. The question is not which latitude, longitude and elevation information should be used (that is clear; the gravity station description information should be used), but rather where the information on the field observation sheet came from and whether the station is really the station it is purported to be. To muddle the situation even further, gravity station description forms for the same station have been received which are identical in description and date except for a change in a coordinate of the station and/or station designation. See Figure 15 and Figure 16 for examples of this situation.

To add even more confusion, some stations do not have a gravity station description form. The most common station of this type is commonly referred to as a drift station [Spita, 1981, private

GRAVITY STATION DESCRIPTION	STATION TYPE Calibration Base	STATION DESIGNATION Great Falls 0
COUNTRY USA	STATE/PROVINCE Montana	CITY Great Falls
LATITUDE 47° 29.12′	LONGITUDE -111° 21.20′	ELEVATION 1119.70m
GRAVITY STATION MARK None	AGENCY/SOURCE	INSCRIPTION
POSITION REFERENCE MAP	POSITION SOURCE USGS 7.5′	SOURCE DESIGNATION Southwest Great Falls, Mt. 1965
ELEVATION REFERENCE MAP	ELEVATION SOURCE USGS 7.5′	SOURCE DESIGNATION Southwest Great Falls, Mt. 1965

POSITION/ELEVATION REMARKS

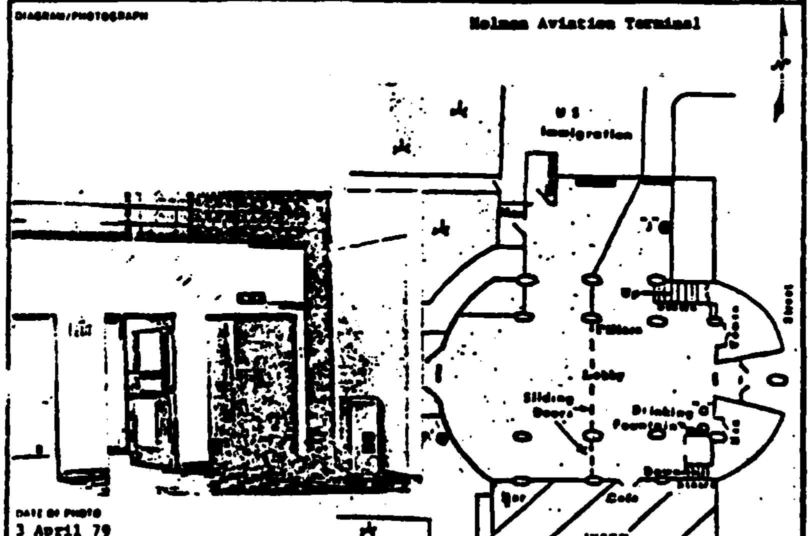
C.I.- 20 feet

DESCRIPTION

Station is west of Great Falls on Gore Hill, northeast of the International Airport Terminal, at the Holman Aviation Terminal (Old International Terminal), inside and south of the main entrance just south of the entrance to the men's room, about 2 meters north of the stairs leading to the basement next to the pillar and east of the drinking fountain along the wall.

IGB: 156710 USAMSFE: Great Falls Muni Ap

DIAGRAM/PHOTOGRAPH



DATE OF PHOTO

3 April 79

DESCRIBED/CONTROLLED BY	AGENCY	DATE
Grabowski	DMAHTC/GSS	3 April 79

DMATC FORM 8250-9 (GSS)
OCT 77

Supersedes GSSQ FORM 8250/OD-9, May 73 which is obsolete.

Figure 15 — Gravity station description form for station "Great Falls 0" — sample no. 1.

GRAVITY STATION DESCRIPTION	STATION TYPE Calibration Base	STATION DESIGNATION Great Falls O
COUNTRY USA	STATE/PROVINCE Montana	CITY Great Falls
LATITUDE 47° 29.00'	LONGITUDE -111° 22.00'	ELEVATION 1119.70m
GRAVITY STATION NAME None	AGENCY/SOURCE	INSCRIPTION
POSITION REFERENCE	POSITION SOURCE	SOURCE DESIGNATION
ELEVATION REFERENCE	ELEVATION SOURCE	SOURCE DESIGNATION

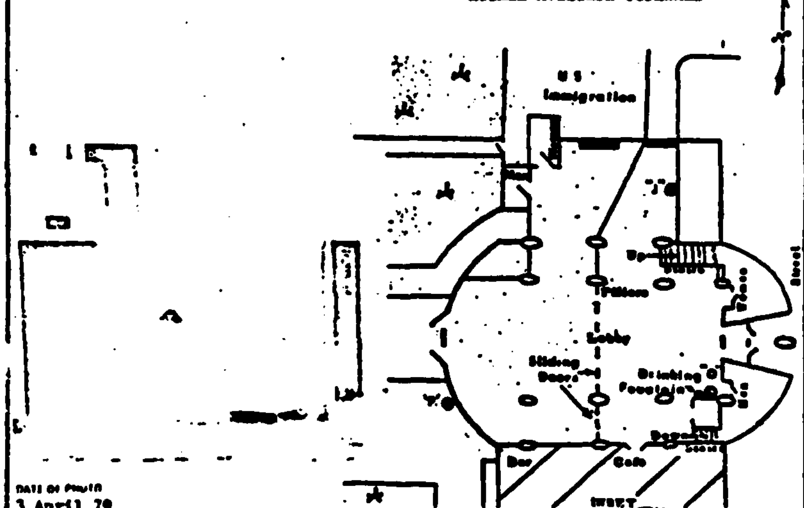
POSITION/ELEVATION REMARKS

DESCRIPTION

Station is west of Great Falls on Gore Hill, northeast of the International Airport Terminal, at the Holman Aviation Terminal (Old International Terminal), inside and south of the main entrance just south of the entrance to the men's room, about 2 meters north of the stairs leading to the basement next to the pillar and east of the drinking fountain along the wall.

IGB: 156710 USAMSFE: Great Falls Muni Ap

DIAGRAM/PHOTOGRAPH



DATE OF PHOTO
3 April 79

	AGENCY	DATE
Grabowski	DMAHTC/GSS	3 April 79

DMAHTC FORM 8250-9 (GSS)
OCT 77

Supersedes GSSQ FORM 8250/OD-9, May 75 which is Obsolete.

Figure 16 – Gravity station description form for station "Great Falls O" – sample no. 2.

communication]. A drift station is generally established when the observer is travelling between assigned gravity station and observer has to make an overnight stop. The observer in this situation will establish the drift station at this overnight stop, making an observation that night and again the following morning, before starting off for the assigned gravity station. The approximate coordinates of this drift station can be obtained from a topographic map of the region but generally no gravity station description for the station is ever produced since the chances are the station will never be reoccupied. It is recommended that for every station where a gravity meter observation is made, there should always exist a gravity station description so the station could be reoccupied.

Another problem is that of the station designation which can be very misleading. A station designation of just a name is not generally enough. For example, there are two stations in the United States Base Station Network by the name of Las Vegas B. One station is in Nevada and the other is in New Mexico. Generally, in addition to a name for a station, an identificiation code such as an International Gravity Bureau (IGB) number is associated with the station. In the case of the station, Las Vegas B, in Nevada, the identification code assigned was 12065B because this station was in the IGSN 71. However, the Las Vegas B station in New Mexico had no identification code on its gravity station description sheet. But on some of the field observation sheets, the station appears with an identification code of 11955B. The matching of gravity station description information with the

information supplied on the field observation sheets can become very confusing. This leads to the real possibility that the same station could be identified in the adjustment as two or more different stations. This possibility would result in a weaker network adjustment and some confusion since there would be two adjusted gravity values for the same station.

3.3.1 Station Identification

In order to uniquely identify stations for the IGSN 71, the IGB, International Gravity Bureau, number or code was established [Morelli, et al, 1974]. The main feature of this coding system was that it conveyed information about the geographical coordinates of the station it was identifying. The IGB code consists of five digits and a letter. The first three digits of the code are determined from the geographical coordinates of a station using the following relationships given in IGSN 71 [Morelli, et al., 1974]. See Figure 17 for how the three digit code is distributed over the earth.

The other two digits of the IGB code are the units of the latitude and longitude degrees respectively. In the formation of these digits, no rounding-off is done.

In order to identify uniquely stations that have the same five digit number, a unique letter is attached to the end of the five digit code. This permits up to 26 stations, one for each letter of the alphabet, to be assigned a unique code for every 1x1 degree block on the earth.

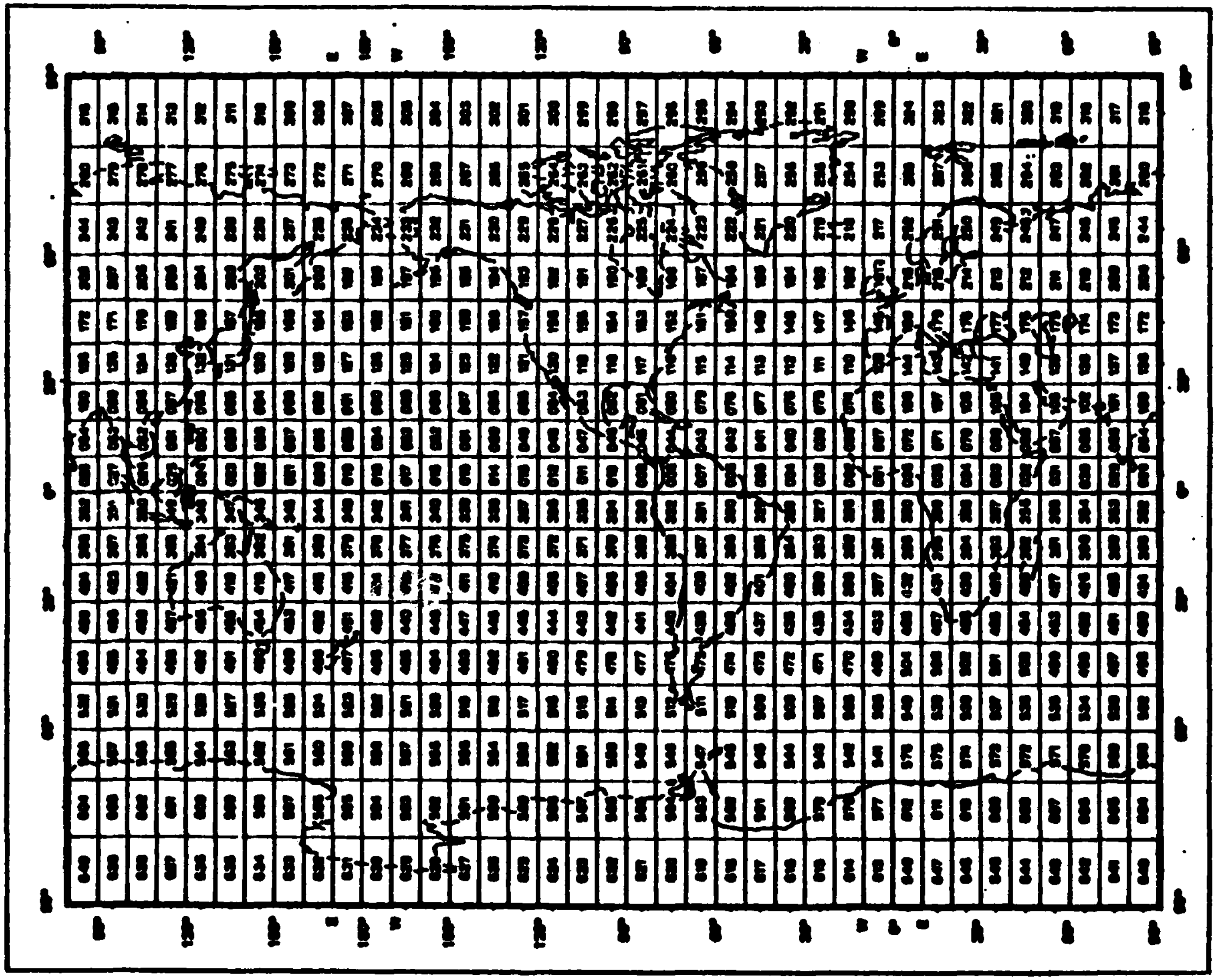


Figure 17 — Distribution of the first 3 digits of the IGB code over the world.

In a world-wide gravity base station network like IGSN 71, 26 stations per 1x1 degree block were sufficient. But, when local base station networks are being established or densified, the number of stations in a 1x1 degree block might exceed the 26 stations permitted when using the IGB code system. In the United States, the U. S. Interagency Gravity Standards Committee recommended a solution to this problem. The solution was to add an additional letter to the end of the IGB code which would permit 676 or more stations to be uniquely identified in any 1x1 degree block. The exact number depended on whether blanks and/or numerals were considered letters [Uotila, 1981, private communication].

From a data management point of view, this change might necessitate a modification in the data base structure for gravity meter observations if the IGB code were being used to identify the stations. This is because the proposed "modified" IGB code would require seven characters as opposed to six characters presently being used.

In order to avoid increasing of the number of characters needed to identify a station and its location, a possible solution would be to have kept a six character code with the first two characters representing the 10x10 degree blocks instead of the first three characters as is done in the IGB number. This could easily be accomplished since there are 26 letters, a-z, and 10 numerals, 0-9, for a total of 36 characters that can be used. With thirty-six 10 degree intervals in longitude and only eighteen 10 degree intervals in latitude, each 10 degree interval in latitude or longitude can be

represented by one of the 36 characters available. This would permit the geographical 1x1 degree information to be represented in four characters instead of five required by the present IGB code. This would leave a set of two characters for station identification within each 1x1 degree block. This coding method would not change the length of the code and relate the same information.

However, if this coding system were used, an existing station would have a new identification code along with its IGB code. This could lead to some additional confusion. But, if the fifth character of the new code were always an alphabetic character, then the two codes would be unique and easily distinguishable because the fifth character of the IGB code is always numeric.

The real concern is not which coding system is adopted, but whether the system will be used by all organizations that collect and distribute gravity base station information. If a uniform coding system is not used, the confusion that can occur with station identification will persist.

3.3.2 Gravity Meter Loops and Trips

A loop is a set of gravity meter observations that starts at a station and generally ends on the same station after a number of other observations have been made at other stations. A loop can require a few hours to several days to complete. The recommended types of loops as given in Land Gravity Surveys [DMAHTC/GSS, 1979] are referred to as ladder, modified ladder, and line sequence. A ladder and modified

ladder sequence loops start and end at the same station. In the ladder sequence, every station is observed as if it were a rung on a ladder and the observer climbs up and down the ladder stopping at every rung to make an observation. In the modified ladder sequence, rungs are skipped on the way up or down the ladder. The modified ladder sequence loop is used when difficult field conditions are encountered. Difficult field conditions, for example, occur when the station to be occupied is in a building which is inaccessible to the observer on the day and/or time the observer tries to occupy the station. The line sequence loop does not end at the starting station but at some other stations. See Figure 18 for examples of the types of loops.

A loop in which the first and last station observed is the same is often referred to as a closed loop. A closed loop can be used to determine if a linear drift exists within an instrument [DMAHTC/GSS, 1979]. It is based on the premise that any difference between repeated observations at the same station is a result of a linear instrumental drift. This, of course, is not the only reason why a difference between repeated observations at the same station might exist. One possible reason is that one or more tares could have occurred between the times of the repeated observations which could result in a difference in the observations. Another possible reason is that the difference is due to observational error and not a linear drift.

Determining the presence of a linear drift in an instrument does not require observations to be made following a closed loop structure such as the ladder or the modified ladder sequence. The line sequence loop

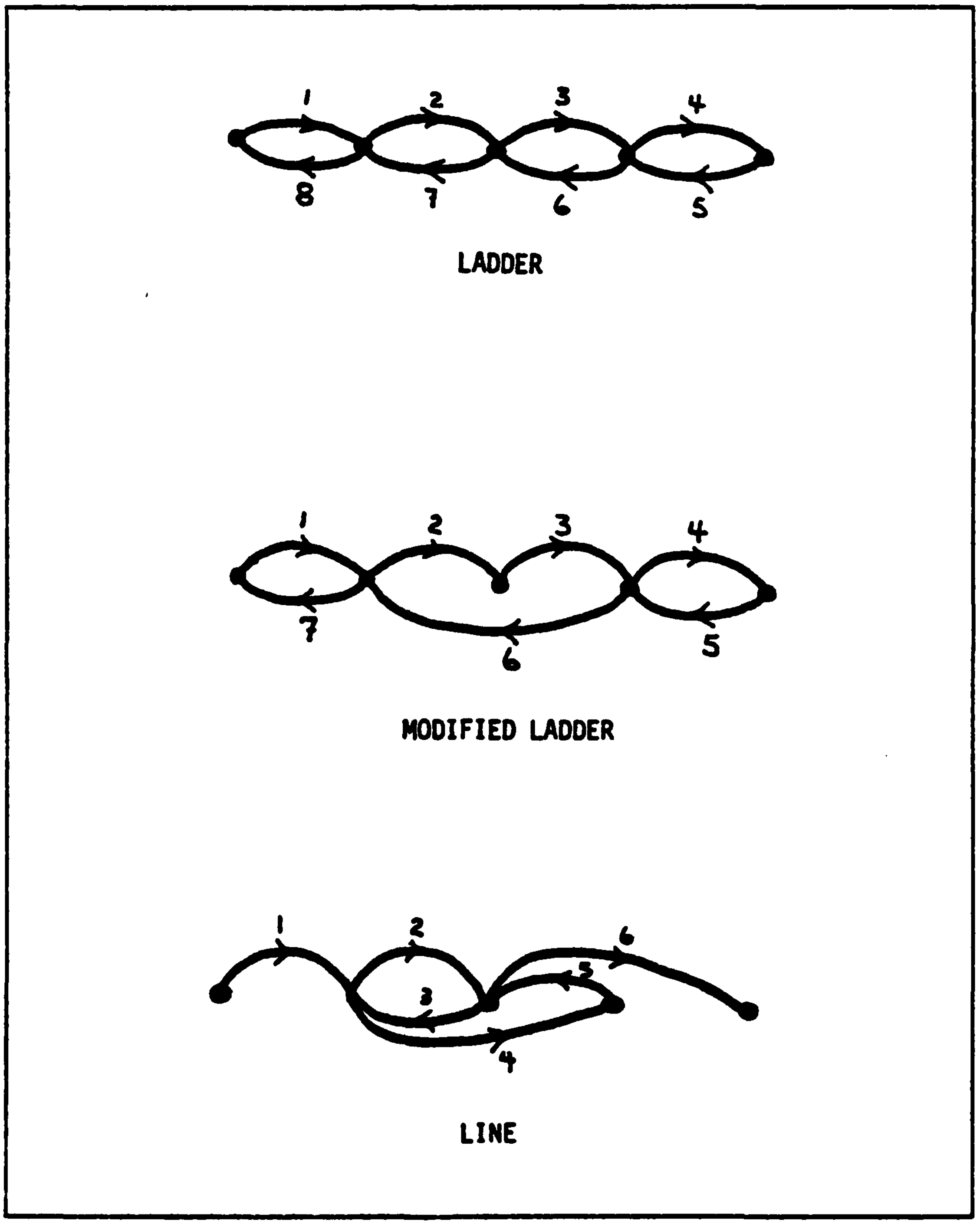


Figure 18 — Types of gravity meter loops possible.

will work just as well. What is necessary is that the relationship for the drift of an instrument should be included in the mathematical model used to describe the instrument's behavior. The mathematical model for the instrument's behavior depends on the two observations used in forming the equation and not on what type of loop the observations were made.

A trip will be defined as a set of consecutive observations for which the gravity meter's behavior is assumed to remain the same. A trip could involve a number of loops or just part of a loop depending on how the instrument is used and what has happened to it. The question arises as to when a trip begins and when it ends. The answer involves the determination of when changes in the behavior of the gravity meter can be expected. There are two good examples of when the behavior of the gravity meter might change. First and the most common is when a tare is introducted into the gravity meter. A tare reflects a discontinuity in the behavior of a gravity meter and cannot be modelled into the observational difference between two consecutive observations because its magnitude and direction are unknown. A number of conditions can result in tares. For example, tares can occur when the gravity meter is taken off—heat and put back on—heat, when the gravity meter balance beam is not clamped during transportation, when the gravity meter experiences a rapid acceleration and/or deceleration such as when the meter is accidentally jarred even if the balance beam is properly clamped and even when the measuring screw is being lubricated [Perry, 1980, private communication].

The other condition under which a new trip should be started is when the activity of the gravity meter is unknown. If the activity of the instrument is not known for a period of time, too many possible things could have occurred to make the gravity meter's behavior inconsistent during that period of time. For example, the meter could have been taken off-heat.

To eliminate personal bias from being introduced when the observations are being divided into trips, a set of rules was used to define what consitutes a trip for a gravity meter. The rules followed were:

1) A new trip will begin with the first observation after a tare has been detected.
2) A new trip will begin when the time between consecutive observations exceeds T1 hours.
3) A new trip will begin when the time between consecutive observations at the same station exceeds T2 hours.

The value of 24 hours was assigned to T1 and a value of 6 hours was assigned to T2. The reason for choosing the value for T1 is that if a gravity meter was being used to make observations in a loop, one would not expect the time between observations to be longer than a day before the loops was completed. The choice for T2 is based on the premise that repeated observations at the same station would only occur when the observations made were part of the same loop. This occurs, generally, at overnight stops. Then it would be expected that the repeated observations occurred the night before and the next morning,

which generally meant a time interval of 6 hours or more. Therefore, repeated observation at the same station separated by more than 6 hours would indicate that the latter observation would be the first observation of a new loop.

To verify this reasoning, an extensive study was performed to see what the effect would be of dividing the observations into trips based on various length of time. The results of this study indicated that when the value of T1 was increased, the apparent accuracy of the observed counter readings decreased. This was due to tares being included which were present in the increased time interval previously ignored. Conversely, as the value of T1 decreased, the accuracy of the observed counter readings in some cases increased to unrealistic accuracies of less than 10 μgal when T1 was set to 4 hours. This occurred because generally only small gravity differences could be observed in that time interval.

The study also looked at the possibility of the gravity meter having a different behavior during the overnight stops as opposed to the normal observation sequence. However, due to the limited number of overnight differences available for each gravity meter, there was not sufficient evidences that any change in the gravity meter's behavior occurred.

3.4 Observational Errors

As mentioned previously, errors such as tares can easily be introducted into gravity meter observations. There is little that the

erver can do to guard against these type of errors, except handle gravity meter with care when it is being transported. However, e errors can be introduced into the observations no matter how much e is taken in its transportation. These errors are due to improper ration and adjustment of the gravity meter itself.

The gravity meter should not be without power for any length of time ause without the power the operating temperature of the gravity er cannot be maintained. To insure a stable operating environment, gravity meter must be kept on-heat. Therefore, the gravity meter uld always be connected to either a battery or the rger-eliminator except for the short time required to change between two power supplies.

As mentioned previously, transporting the gravity meter should only ur when the beam is clamped. Failure to follow this simple rule l, almost surely, introduce tares into the gravity meter.

The proper adjustment of the gravity meter is also very important to ure the stable behavior of the meter. The two gravity meter levels, long level, (parallel to the counter), and the cross level, rpendicular to the counter) must be adjusted according to the rating manual provided with each gravity meter [LaCoste & Romberg, 0]. The quality of the observations made with a gravity meter ends to a large part on these two levels being in proper adjustment. deviation from the correct position of the levels will change the vity meter's sensitivity and reading line.

The sensivitiy should be checked periodically to assure that the recommended sensitivity is kept at 8 to 12 eyepiece divisions for every dial revolution [LaCoste & Romberg, 1980]. If the sensitivity is not kept within this range, the reading line for the instrument will change. For a decrease in the sensitivity, the reading line will shift up the scale. Conversely, for an increase in the sensitivity, the reading line will shift down the scale [DMAHTC/GSS, 1979].

Even if the instrument is in perfect adjustment, observational errors can be introduced by the observer through the nulling of the instrument and the reading and recording of the observation.

3.5 Honkasalo Correction Term

The so called Honkasalo correction term is a latitude dependent correction which was applied to all absolute gravity sites used in the IGSN 71 adjustment [Morelli, et al., 1974]. The result was that all adjusted gravity station values published for the IGSN 71 included the Honkasalo correction term. The correction is based on the premise that the earth tide correction applied to measured gravity given by equation (1) in Honkasalo [1964] is only zero when summed over the whole earth's surface and not zero when summed over a particular latitude. This systematic effect according to Honkasalo [1964] should be removed if the earth tide correction summed over a particular latitude is to be zero. The amount to be removed is given by equation (5) in Honkasalo [1964].

At the XVII General Assembly of the International Union of Geodesy an Geophysics at Canberra, 2-14 December, 1979, Resolution No. 15 was passed by the International Association of Geodesy which resolved that the Honkasalo correction should not be applied to observed gravity [Uotila, 1980]. Therefore, all published IGSN 71 station values should have the Honkasalo correction removed according to the method given by Uotila [1980].

In this regard, care must be taken to assure that any station value used as control in an adjustment does not include the Honkasalo term. The Italian absolute station determinations as given in Marson and Alasia [1978] include the Honkasalo correction term. In order to have a consistent set of absolute stations, the Honkasalo term must be removed from those absolute station values since the later Italian determinations [Marson and Alasia, 1980] and all U. S. determinations made by Hammond do not include the Honkasalo term [Hammond, letter to Uotila, 1981].

3.6 Why The Gravity Value Changes

Environmental and geophysical changes can result in the actual changing of the value of gravity at a site. These changes can be classified as either long term which tend to be of a permanent nature and short term which tend to be of a temporary nature.

3.6.1 Long Term Effects

Long term variations in the value of gravity at a station result from geophysical changes within the earth. These changes are caused by a redistribution of the earth's masses resulting in a change in the value of gravity at a station. Some examples of effects that can cause these long term variations are: displacement of the earth's core, mass redistribution in the crust and/or mantle, changes in the position of the station, changes in the earth's rotation and/or figure and changes in the gravitational constant [Boedecker, 1981]. These long term variations, generally, are caused by geophysical events having unpredictable effect on the value of gravity at a station. For this reason, the modelling of these effects in a gravity station network is presently not feasible. The magnitude for these types of effects may be on the order of tens of μgal/year [Boulanger, 1979].

3.6.2 Short Term Effects

Short term variations in the value of gravity at a station results from such things as earth tide, variation in the level of the groundwater, and changes in the distribution of the atmospheric masses [Boedecker, 1981] are more predictable than the long term variations previously mentioned, provided sufficient data is available. The earth tide which is caused by the gravitational pull of the moon and sun on the earth can be theoretically be modelled to an accuracy of better than 0.01 μgal [Heikkinen, 1978] provided adequate information is available about the location of the station and the epoch of the

observation. The change in the level of groundwater below a station results in a predictable change in the value of gravity. This is the same as changing the mass distribution below the station. To determine the effect that changes in the groundwater levels has on the value of gravity at a station requires detailed information about the extent and level of the groundwater at the epoch of the gravity observation. This information is generally not available. This makes modelling the effect of the changes in the groundwater level not feasible. The magnitude of the change in gravity at a station caused by a change in the ground water level is generally in the range of 10–20 μgal but has been reported to be more than 100 μgal [Boulanger, 1979].

The short term variations in the value of gravity caused by changes in the atmospheric masses above a station is seasonal in nature with estimates of this variation being as large as 20–30 μgal [Boulanger, 1979]. Even though this variation might be able to be modelled, its effect on the gravity difference between two consecutively observed stations could probably not be detected since the time between gravity observations is, generally, less than six hours while the change in the mass distribution of the atmosphere is assumed to be more gradual, taking on the order of days to weeks before changes in the value of gravity can be observed. This effect should be considered when absolute gravity measurements are being made.

Besides short term variations in the value of gravity at a station, there are short term effects that influence the observations made with a gravity meter. These are caused by variations in the voltage,

temperature, atmospheric air pressure, and magnetic field. The effect these variations have on the gravity meter observations depends on the individual instrument being used. Under controlled conditions, the effect of each appear to be predictable [Kiviniemi, 1974]. However, the magnitude and direction of the effect requires additional information, such as the voltage of the power supply, atmospheric pressure, temperature, and alignment of the gravity meter relative to magnetic north, which is not generally recorded when the gravity meter observations are made.

CHAPTER FOUR

GRAVITY BASE STATION NETWORKS

4.1 Introduction

A Gravity Base Station Network consists of a set of recoverable gravity stations distributed over a geographical region for which the value of gravity has been determined. There are two methods available for determining the value of gravity of a station in a network. One method of determining a station's gravity value is by direct measurement of gravity. This is done by instruments referred to as absolute gravity measuring apparatuses or absolute gravity meters. There are two types of absolute gravity meters, permanent and transportable (portable) which either employ the free fall or the symmetrical free rise and fall technique [Sakuma, 1976]. With the permanent absolute gravity meters, claims for their accuracies or precision on the order of a few μgal are made; while with the portable absolute gravity meters, accuracies or precision in the neighborhood of 10 μgal are obtainable [Marson and Alasia, 1978; Marson and Alasia, 1980; Wilcox, 1980].

The other method available is by making relative gravity meter ties from stations of known gravity values to other stations. The most common gravity meter used in the geodetic community for this purpose

are the LaCoste & Romberg 'G' and 'D' gravity meters. See Section 1.2 for a description of the basic difference between the LaCoste & Romberg 'G' and 'D' gravity meters.

The portable gravity meters, such as LaCoste & Romberg 'G' gravity meters, provide only relative gravity difference information about a network and nothing concerning the actual gravity value of a station. The absolute gravity meters on the other hand provide information about the actual value of a station. Once a station's gravity value is known, portable gravity meters can be used to make ties between stations with known gravity values and stations with unknown gravity values. However, the gravity difference can only be deduced if the relationship between the counter unit difference and their actual gravity difference is known. The relationship is purported to be given by the Calibration Table 1 supplied with each gravity meter. As it has been mentioned, the scale for this calibration table comes from an assumed gravity difference between two stations in New Mexico, Cloudcroft and La Luz. If the assumed gravity difference between these two stations is in error, then the scale factor determined from their assumed difference would cause the Calibration Table 1 to be off by a scale factor.

To determine the scale factor that is to be applied to Calibration Table 1 requires the knowledge of at least one gravity difference. This can only come from the difference between two stations of known gravity value. A station of known gravity value is often referred to as an absolute station. Having more than two absolute stations does

not necessarily provide more information concerning the linear scale factor that is being attempted to be determined. The distribution of the known gravity stations over the range of the network plays a very important role in how well the scale factor can be determined [Uotila, 1978]. In order to obtain the best determination of the linear scale factor requires that interpolation instead of extrapolation be done. In addition, the shorter the interpolation interval, the better. It is clear that for a network having a gravity difference between the gravity station with the largest gravity value and the gravity station with the smallest gravity value of X mgal and consisting of n known gravity stations, the scale factors are best determined when the gravity difference between known gravity stations is approximately $X/(n-1)$ mgal with $n \geq 2$.

4.2 Control of Network

If the control for a network is not good, then the results of the network adjustment can not be expected to be good. The absolute gravity stations in a network provide the control for the network. In the U. S. Gravity Base Station Network, two different absolute gravity measuring devices were used. One was from Italy [Marson and Alasia, 1978; Marson and Alasia, 1980] and the other was from the United States [Hammond and Iliff, 1978]. The accuracy of the determination made with each of the instruments was purported to be in the neighborhood of 10 μgal. But the difference between the values of gravity determined at the same station by these instruments has on occasion been as large

as 100 μgal or more [Marson and Alasia, 1978 and 1980; Hammond, 1981, letter to Uotila]. The reason for such large differences between the determinations made by these instruments is unknown. Even more puzzling is why repeated determinations made with the same instrument do not agree very well. By comparing the values in Table 4 and Table 5, examples of differences between gravity values at the same site can be seen. situations.

A variety of reasons can be hypothesized why the two absolute gravity measuring devices give different values of gravity at the same station. Some reasons are: there is a systematic difference between the two instruments; there is a scale problem with the timing and/or distance required for the determination; the gravity value at the station actually changed; and external forces influenced the determination.

Of these possible reasons, only the last one has been proven to be a real cause. When the Italian apparatus made measurements at the Holloman, AFB in New Mexico in May and June of 1980, difference between the two determinations of the value of gravity at the site of 80 μgal was noticed. Further investigation as to the reason for this difference revealed that the gravity value obtained depended on whether a gyro testing system in a near by building was operating. When the gyro testing system was not in operation, the gravity values determined by the Italian and the United States absolute gravity apparatus agreed very well. However, when the gyro testing system was operating, the values determined disagreed by approximately 80 μgal. It seems that

Table 4 — Listing of values of absolute sites determined by Marson and Alasia.

IGB/GSS Code	Station Name	Gravity in μgal	Standard Error in μgal	Date Determined
15221A	Boston,MA	980378659*	11	10/ 8–11/77
11994H	Denver,CO	979598267*	10	10/16–19/77
15560A	Bismarck,ND	980612882*	10	10/25–27/77
11926A	Holloman,NM	979139513*	10	11/ 3–7 /77
12172A	San Francisco,CA	979972060*	10	11/15–17/77
08150C	Miami,FL	979004319*	10	11/21–26/77
15505D	Boulder,CO	979608498	11	5/26–27/80
11926A	Holloman,NM	979139584	12	6/ 2–3 /80
119S04	McDonald Obs.,TX	978820097	11	6/ 6–7 /80
155V04	Sheridan,WY	980209007	11	6/12–14/80
156E05	Great Falls,MO	980497412	10	6/17–18/80
231A01	Anchorage,AK	981928998	10	6/27–28/80

IGB — International Gravity Bureau
GSS — Geodetic Survey Squadron

* denotes Honkasalo correction removed from published value.

Information obtained from [Marson and Alasia, 1978 and 1980].

Table 5 — Listing of values of absolute sites determined by Hammond.

IGB/GSS Code	Station Name	Gravity in μgal	Standard Error in μgal	Date Determined
11994H	Denver,CO	979598277	10	3/27–29/79
119S03	McDonald Obs.,TX	978828655	8	7/ 3–4 /79
11926A	Holloman,NM	979139600	10	7/ 6–7 /79
119C01	Trinidad,CO	979330370	10	7/10–11/79
119C03	Mt. Evans,CO	979256059	8	7/12–13/79
155V01	Casper,WY	979947244	25	7/15–17/79
155V03	Sheridan,WY	980208912	10	7/18–19/79
156E05	Great Falls,MT	980497311	10	7/21–22/79
11926A	Holloman,NM	979139600	8	5/14,31/80
15221A	Boston,MA	980378681	10	7/ 7 /80
156E05	Great Falls,MT	980497367	10	10/ 9–11/80
155V03	Sheridan,WY	980208964	10	10/13–16/80
15505D	Boulder,CO	979608601	10	10/18–23/80
119C01	Trinidad,CO	979330393	10	10/25–26/80
119S04	McDonald Obs.,TX	978820087	10	10/28–29/80
15221A	Boston,MA	980378768	10	2/ – /81

IGB — International Gravity Bureau
GSS — Geodetic Survey Squadron

Information obtained from [Hammond, 1981, letter to Uotila].

the electronics of the Italian apparatus were affected by the gyro testing system while the United States apparatus' electronics were not affected [Wilcox, 1980].

Any errors that exist in the value of gravity at the absolute stations will be reflected directly into any scale factors for the gravity meters determined when the network adjustment is performed. As a result, the value of gravity determined for the other stations in the adjustment will be affected.

The quality of a gravity network not only depends on the accuracy of the absolute stations' gravity values but also on the distribution of the absolute stations in the network and the gravity meter ties made between gravity stations. The question that comes up is what is the best network configuration for a set of gravity stations.

4.2.1 Criteria for the Best Network

In order to say that one network is better than another, a criteria must be established which will enable this decision to be made. Assuming there are two networks, E_1 and E_2, each containing the same stations but with different gravity meter ties made between the stations and possibly different absolute gravity station, a decision as to which one is preferred, based on the variance-covariance matrices for their adjusted station values, can be made using one of the properties described by Fedorov [1972].

Uotila [1978] points out that the most appropriate criteria for comparison of gravity base station networks involves the

variance–covariance matrix for the adjusted station values which has the minimum trace. The trace of a matrix is the sum of its diagonal elements. Therefore, one can say that network, E_1, is preferred to network, E_2, if the trace $\sum_{E_1}$ is less than the trace $\sum_{E_2}$.

Using this criteria, Uotila [1978] describes a method for determining at which station in a network an absolute gravity measurement should be made in order to improve the network the most. This method can be used with a slight modification for determining which gravity meter tie would improve the network the most.

4.2.2 Selection of the Gravity Meter Tie to Improve a Network

Since a gravity meter tie provides information about the gravity difference between existing stations in the network, the best gravity meter tie to make would be the one which results in the biggest improvement in the variances for the station gravity values. The selection can be made by using a slight modification of the method Uotila [1978] described for selection of absolute gravity sites.

Assuming that the minimum variance solution for a set of equations for a gravity base station network is given by

$$X = N_1^{-1}U \tag{4.1}$$

and

$$X_a = X_o + X \tag{4.2}$$

where

X_o — initial estimated value of parameters,

X — correction to X_o,

X_a — adjusted parameters values,

U — constant vector of the normal equations,

N_1^{-1} — variance—covariance matrix of the adjusted parameters,

ien, if a set of observations, L_b^2, with their variance—covariance itrix, $\sum_{L_b^2} = P_2^{-1}$, and a mathematical model

$$L_b^2 = F(X_a) \tag{4.3}$$

e added to the original solution, the combined solution for the rameters is given by

$$X_a^2 = X_o + X_2 \tag{4.4}$$

ere

$$X_2 = -(N_1 + A_2^t P_2 A_2)^{-1}(U + A_2^t P_2 L_2) \tag{4.5}$$

d

$$A_2 = \left.\frac{\partial F_2}{\partial X_a}\right|_{X_a = X_o} \tag{4.6}$$

$$L_2 = L_o^2 - L_b^2 \tag{4.7}$$

$$L_o^2 = F_2(X_o) \tag{4.8}$$

AD-A120 488 MATHEMATICAL MODELLING OF THE BEHAVIOR OF THE LACOSTE AND ROMBERG 'G' GRA..(U) OHIO STATE UNIV COLUMBUS DEPT OF GEODETIC SCIENCE AND SURVEYI.. L A KRIEG NOV 81 321 2/2

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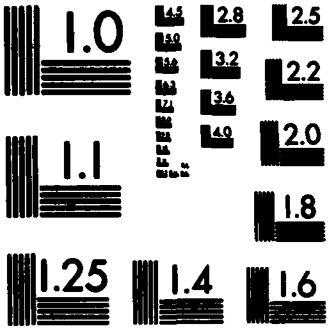

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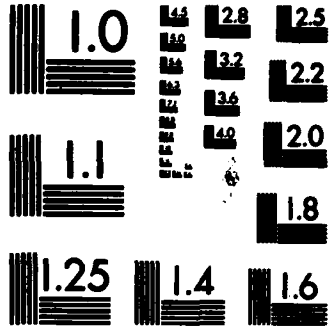

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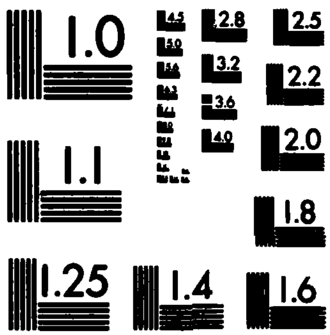

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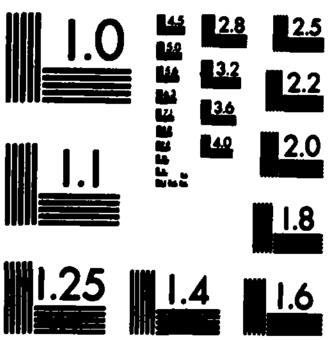

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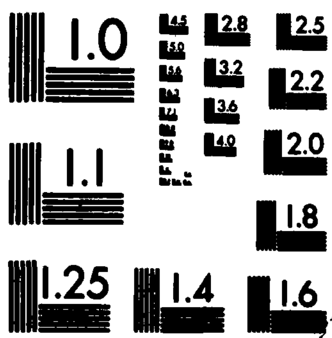

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As shown by Uotila [1978b], the variance-covariance matrix of the adjusted parameters for the combined solution is given by

$$\sum_{X_a^2} = N_1^{-1} - N_1^{-1}A_2^t(A_2N_1^{-1}A_2^t + P_2^{-1})^{-1}A_2N_1^{-1} \qquad (4.9)$$

The change in the trace of the variance-covariance matrix for the adjusted parameters resulting from the including of the new gravity difference information will always be negative and its magnitude is given by

$$N_1^{-1}A_2^t(A_2N_1^{-1}A_2^t + P_2^{-1})^{-1}A_2N_1^{-1} \qquad (4.10)$$

What is desired is the gravity difference which will make (4.10) the largest. Assuming that a single uncorrelated gravity difference, g_{ij}, can be observed with a variance of $\sigma^2_{g_{ij}}$ between two station whose gravity values are given by g_i and g_j where

$$g_{ij} = g_i - g_j \qquad (4.11)$$

then A_2 matrix will be a row matrix of zero elements except for a +1 in the i-th column and a -1 in the j-th column. The value of

$$A_2N_1^{-1}A_2^t + P_2^{-1} \qquad (4.12)$$

can be shown to be the value of the variance for the i-th and j-th station minus twice their covariance plus the variance of the new observed gravity difference. The inverse of (4.12) is the reciprocal of that sum. The matrix product of $A_2N_1^{-1}$ will result in a matrix formed from two columns of N_1^{-1}, one being the i-th column and the other

being the negative of the j-th column. If the two columns of the matrix, $A_2N_1^{-1}$, are thought of as the vectors, Y_i and Y_j, then the value given by (4.10) can be shown to be the value of

$$\frac{Y_i \cdot Y_j}{\sigma_i^2 + \sigma_j^2 - 2\sigma_{ij} + \sigma_2^2} \tag{4.13}$$

where

$\cdot$ - represents the dot product of two vectors,

σ_{ij} - covariance between station g_i and g_j,

σ_2^2 - variance of the new observed gravity difference.

Therefore, the gravity difference between stations g_i and g_j which maximizes the value of (4.13) would indicate where a gravity tie should be made to improve the network the most. This method is similar to the one described by Uotila [1978b] for selection of absolute gravity sites.

In a network of n stations, there is a possibility of n(n-1)/2 different gravity differences with at least n-1 of these differences already existing in the network. By using this method, the effect on the trace of the station's variance-covariance matrix of adding a new and/or existing gravity meter ties with a certain accuracy can be seen.

For every station added to a network of n stations, n more possible gravity differences are introduced. Many times these stations which are added to a network are very close to an existing station. The added stations will generally have gravity values very close to that of the neighboring station that had been previously established. As a

result, the small gravity differences, generally less than 10 mgal, provide little additional information that can be used in improving the determination of the gravity meter scale factor. These types of stations are referred to as eccentric stations. If there is a large number of eccentric stattons in an area, any improvement in the area will result in a large improvement in the trace of the stations' variances, but in reality the actual improvement of the variances for stations outside the local area could be small. In such a situation, a local improvement results rather than the desired over all network improvement.

Therefore, when attempting to select where new gravity meter ties should be made, if possible, eccentric stations should not be considered. If possible, only one station in a local area should be used. The station with the most gravity ties to stations outside the local area should be used, provided that all the eccentric stations in an area are adequately tied to each other.

CHAPTER FIVE

MATHEMATICAL MODELS

5.1 Types of Observables

A gravity meter reading involves the observer recording the value of the counter and the position of the dial after the gravity meter has been nulled, remembering that 1 counter unit is equivalent to 100 dial units. The observation is then generally converted to its value in milligal by interpolating within the factory supplied Calibration Table 1. The manufacturer suggests that a linear interpolation be made within the Calibration Table 1 to obtain the value in milligal for an observed reading [LaCoste & Romberg, 1980]. This can be accomplished by following the simple procedure outlined. First, the value in milligal for the counter reading which is nearest to, but less than, the observed reading is determined. Let the counter reading value used be Y and its corresponding value in milligal be X. The difference, Z, between the observed reading and the counter reading used, Y, is then obtained. Next, multiply the difference, Z, by the factor for interval for the counter reading, Y, and add the result to the value of milligal, X. The result is the value of milligal for the observed reading.

Using a linear interpolation procedure within the Calibration Table 1 assumes that the relationship between the counter readings and the values in milligal can be adequately represented by a piece-wise linear function. Since the Calibration Table 1 information is produced by reading the value of factor for interval off of a continuous curve of factory calibration scale factor data, it is possible that a piece-wise continuous relationship, such as a cubic spline [Spath, 1964], should be used. When a comparison was done to see the difference between the use of a linear or cubic spline interpolation procedure within the Calibration Table 1, the maximum observed difference between the two methods was on the order of 2 to 3 μgal. This shows that the behavior of the Calibration Table 1 is relatively smooth.

Since the Calibration Table 1 attempts to represent a continuous type of continuous information, the cubic spline method of interpolation would be preferred to the linear method of interpolation even though the difference between the two methods is small.

As mentioned previously, the values in milligal are derived from the factor for interval values based on the assumption that the factor for interval over a particular interval, usually 100 counter units, is constant. In reality, this is not exactly true. The factor for interval values represent a continuous function as opposed to a piece-wise linear function. Therefore, the values in milligal should be determined from the integration of this continuous function or an approximation of it, such as a cubic spline. The difference between

the Calibration Table 1 values in milligal and those computed from a cubic spline representation of the factor for interval can be as large as 10-15 μgal over a 300 mgal interval. The large differences can lead to a systematic error being introduced in large gravity differences, over 100 mgal, which would not be evident in smaller differences. This means that as the gravity difference increases in size the variance of the difference about the mean of the gravity difference that is acceptable should also be increased. This makes the detection of tare of a given size more difficult for gravity differences that are large as opposed to those that are small.

Once the value in milligal for all readings has been determined, the gravity difference between two stations can be determined by computing the difference between their values in milligal. However, the resulting difference will not be independent of the time of the observation and might have to be scaled by a factor to obtain the proper units of gravity. The difference can be made time independent by removing any known time dependent effect such as the earth tide effect. In order to obtain a gravity difference in the proper units, the absolute scale factor applied to create the Calibration Table 1 must be correct and valid for the range in which the gravity meter was being used. It must be remembered that the absolute scale factor applied to the relative scale factor during the factory calibration procedure was truly valid only over a range of approximately 242 mgals for gravity values in the region of 979 gal. Assuming it to be valid for any range over which the gravity meter is being used, might not be

correct.

It is clear that the only true observed quantity for a gravity meter is the observed counter and dial reading. From the observed counter readings, their values in milligal are derived. And finally, the gravity difference between two values in milligal can be obtained.

5.1.1 Observed Counter Reading

The quantity that is observed on a gravity meter is the position of the counter and the dial which when combined yields the observed counter reading. The problem with using this quantity as the observable in a mathematical model is finding an analytical relationship that will tranform the counter readings into units of gravity. An empirical relationship exists in the form of the Calibration Table 1 which is supplied with each LaCoste & Romberg 'G' gravity meter. It is necessary to determine if there is an analytical expression for the empiricial relationship expressed in the Calibration Table 1 which can be used as a functional relationship in a least squares adjustment model. If not, it is necessary to determine if there is some analytical function which approximates that empirical relationship. If the relationship can be expressed adequately, then the observed counter readings can be used directly as observables in a least squares adjustment model.

Assuming that such an analytical relationship can be found and that it is a simple function, then solving for periodic screw effects becomes possible without getting into the problem of having to assign

weights to two different observables, counter readings, and their values in milligal which represents the same quantity.

5.1.2 Value in Milligal

If the observed counter reading cannot be used as observables in the adjustment due to the lack of an adequate model for the Calibration Table 1, then the next best quantity that could be used would be the values in milligal for the observed counter readings. A model which uses as its observable, the value in milligal, has been developed by Uotila [1974] and used with some success. The model which Uotila [1974] proposed used an equation involving the difference between two gravity meter observations which implies the following expression for each gravity meter observation

$$\sum_{i=0}^{n} D_i x^i + k(t - T_o) + S - G = 0 \qquad (5.1)$$

where

D_i – coefficient of the i-th order scale factor term,

x – value in milligal of the observed counter reading corrected for all known systematic effects, such as, earth tides an height of instrument above the station,

k – coefficient of the drift term,

t – epoch of the observation,

T_o – some arbitrary initial epoch associated with the set of gravity meter observations,

S — an unknown offset value that must be added to obtain the correct absolute gravity value for the station,

G — gravity value of the station,

n — largest order scale factor term to be included in the model.

Because the linear scale factor term in addition to scaling the value in milligal would also scale all of the systematic effects, this model is theoretically incorrect even when $n = 1$. However, this effect would be minimal because the value of the linear scale factor term is very close to 1. For $n > 1$, the question of what the model represents becomes very confusing. As noted by Uotila [1974], higher order scale factor terms might not have any physical justificiation. But, even if they were physically justified, a similar problem exists when $n > 1$ as when $n = 1$. If x is expressed as $z + C$ where z is the value in milligal of the observed counter reading obtained from the Calibration Table 1 and C is all associated systematic effects, then when $n = 2$, for example, terms involving D^2C and C^2 enter into the model. Theoretically, these terms are not appropriate; however, their affect would be small compared to the value of z^2.

A more appropriate expression for equation 5.1 might be

$$\sum_{i=0}^{n} D_i z^i + k(t - T_o) + S + C - G = 0 \tag{5.2}$$

where the meaning of the symbols has been previously defined. With the expression, the problem of scaling the systematic effects is removed.

5.1.3 Gravity Differences

The most commonly used, and probably most incorrectly used observable is the gravity difference. Many different models have been proposed using the gravity difference as the observable [Whalen, 1974; McConnell and Gantar, 1974; Torge and Kanngieser, 1980]. A typical model might be expressed as

$$l\Delta g_{ij} + k(t_i - t_j) + C_i - C_j - G_i + G_j = 0 \qquad (5.3)$$

where

l — coefficient of the linear scale factor term,

Δg_{ij} — observed gravity meter difference in milligal between station i and j,

k — coefficient of the drift term,

t_n — time of observation at station n,

G_n — gravity value at station n,

C_n — all systematic effects for the observation associated with the observation at n.

Equation (5.3) can be rewritten into true observation form as

$$\Delta g_{ij} = \frac{k(t_i - t_j) + C_i - C_j - G_i + G_j}{l} \qquad (5.4)$$

The observation form exists when the observed quantity is strictly a function of parameters

The least squares solution of this system of equations requires the variance—covariance matrix of the gravity differences computed from observations to be known. Generally, this variance—covariance matrix is assumed to be diagonal which neglects the correlation that exist between successive gravity differences as stated by Uotila [1974].

If the correlation in the variance—covariance matrix of the gravity differences is not included, the effect on the value of the adjusted parameter values will probably be very small. However, the variance—covariance matrix for the adjusted parameters will be greatly affected. As a result, the estimate of the variances for the adjusted parameters will be too low. This results in an over optimistic estimates of the standard error of all parameters.

5.2 Characteristics of Calibration Table 1

Each "G" gravity meter produced is a hand built product which results in its own characteristic behavior. Even though this behavior is different for every gravity meter, the Calibration Table 1 which represents this behavior does exhibit some common characteristics. The functional relationship between the value in milligal and the counter readings, although unknown, is nearly linear. If the linear trend is removed from the Calibration Table 1, the higher order trends can be seen more readily. Attempts have been made to represent the entire Calibration Table 1 by using polynomial functions up to the fifth order with some success [Uotila, 1978b]. Problems arise in the separation of the coefficents of these higher order polynomials, since the

oefficents tend to be highly correlated with each other. Correlations s large as 0.999 and larger are not uncommon. With these high orrelations, the least squares polynomial fit tends to be very ensitive to the data used in making the fit. Since the actual unctional behavior is unknown, using these higher order polynomials to epresent the relationship could very easily lead to fitting the data nstead of the identification of any general behavior characteristics.

.2.1 Possible Models

After the linear trend has been removed from the Calibration able 1, it is apparent that the residuals of the value in milligal xhibit a definite trend, a type of wave or periodic function. To etermine the characteristics of the trend exhibited, a spectral nalysis of the values in milligal versus the counter readings for each 'avity meter's Calibration Table 1 was performed. The analysis idicated that the only sinusoidal term present was a low frequency rm. The wave length of this term was in all cases larger than 4000 unter units. The median value and mode of the wave length for all e gravity meter was around 8000 counter units. See Figure 19 for amples of this trend. One possible relation other than a higher 'der polynomial that could be used to model this type of trend is a nusoidal relation given by

$$B_1 \sin\left(\frac{2\pi d}{T} \right) + B_2 \cos\left(\frac{2\pi d}{T} \right) \qquad (5.5)$$

ere

d — the counter reading,

T — the period of the sinusoid,

B_1, B_2 — the amplitude of the sinusoid.

This relationship can be expressed as a single function by using a basic trigonometric identity giving

$$A \cos\left(\frac{2\pi d}{T} + \omega \right) \tag{5.6}$$

where

A — amplitude of the sinusoid,

d — observed counter reading,

T — period of the sinusoid in counter units,

ω — phase angle for the sinusoid.

To see how well the Calibration Table 1's values in milligal can be represented as a function of the counter readings, a least squares adjustment was performed using a model containing a polynomial and sinusoid terms as given by

$$\sum_{i=0}^{n} l_i d^i + \sum_{j=1}^{k} A_j \cos\left(\frac{2\pi d}{T_j} + \omega_j \right) = y \tag{5.7}$$

where

y — value in milligal from Calibration Table 1,

d — counter reading corresponding to the value in milligal,

A_i — amplitude of the i-th sinusoid in milligal,

T_i — period of the i-th sinusoid in units of d,

ω_i — phase angle of the i-th sinusoid in radians,

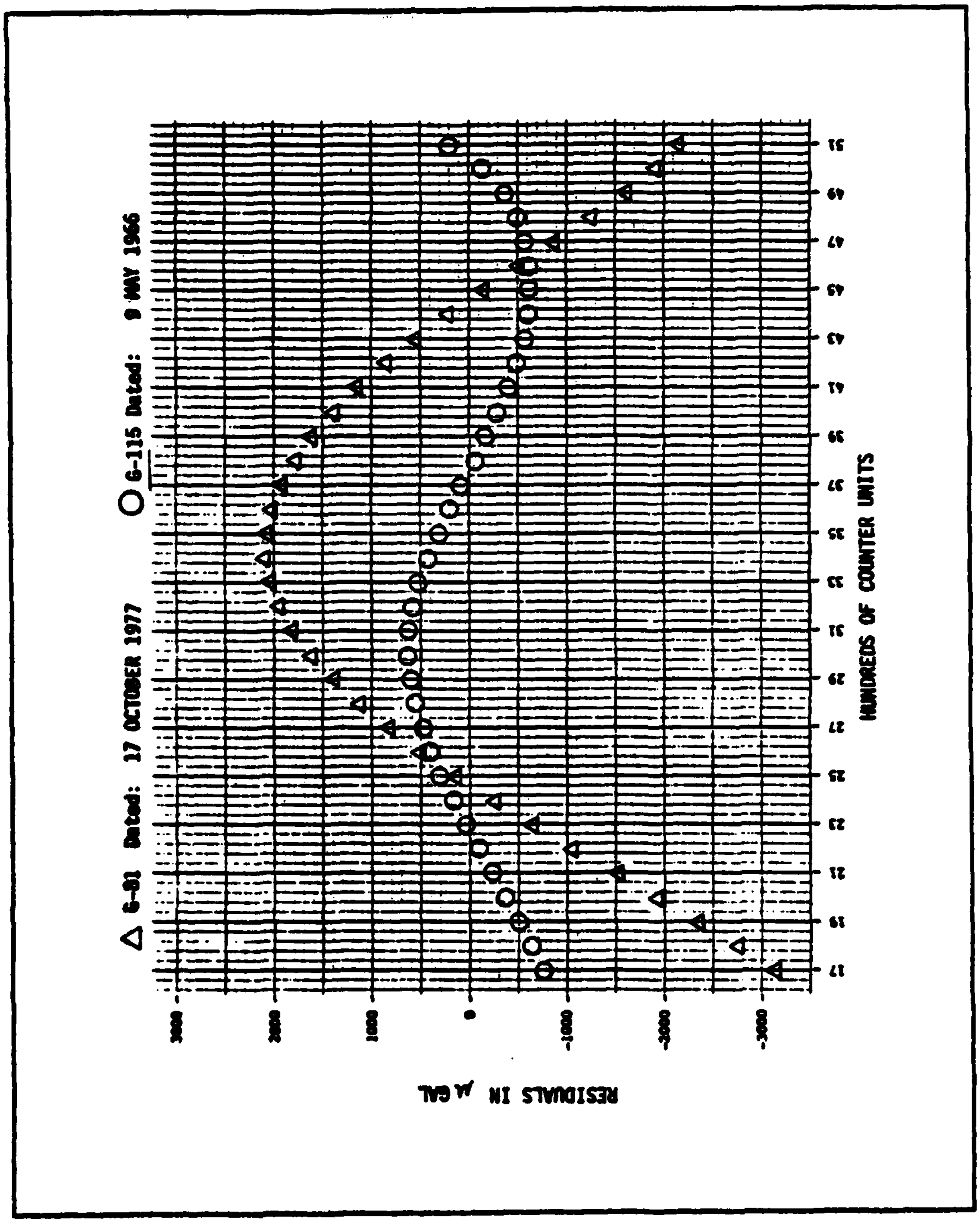


Figure 19 — Plot of residuals after the linear trend has been removed from the Calibration Table 1 for gravity meters G-81 and G-115.

n — the order of the polynomial,

k — the number of sinusoids.

Various values of n and k were selected for a number of counter readings ranges to see how well the Calibration Table 1's values in milligal could be represented analytically as a function of the counter readings. A sample of the results for various models tested is given in Table 6 and Table 7. Table 6 represents the situation in which a gravity meter is used over a world wide range of 5.4 gal with observations made within a range of the counter readings from 700 to 6300 counter units. The uses of a linear model, n = 1 and k = 0, results in root mean square (RMS) fit of the 57 residuals for each gravity meter ranging from approximately 200 to 1200 μgal. However, when a sinusoid was included in the model along with the linear term, n = 1 and k = 1, the RMS fit of the 57 residuals was reduced to a range of approximately 13 to 85 μgal. But for the data used in this study which represents only a limited range of counter, the counter readings from about 2000 to 4400 counter units were used. If only that part of the Calibration Table 1 is used, then the situation gets better as is shown in Table 7. The number of residuals used to produce the results given in Table 7 was 25.

Table 7 compares four different models, three different polynomial models and a sinusoid or often referred to as periodic model. As can be seen in Table 7, for every Calibration Table 1 selected, except for gravity meter 'G-157', the sinusoid model, n = 1 and k = 1, resulted in the best RMS fit of the data. In most cases, the difference between the

Table 6 – Summary of the results of modelling the Calibration Table 1 counter readings from 700 to 6300 using a linear function and a linear function plus a periodic term.

Gravity Meter	Calibration Date	Root Mean Square Fit in µgal Linear	Periodic
G–10	25/10/60	1192.28	41.02 (7650)
G–44	25/04/62	633.31	32.33 (6351)
G–68	23/03/64	755.22	31.00(10222)
G–81	07/08/64	1136.44	16.68 (8430)
G–103	07/09/65	655.67	16.64 (6832)
G–103	14/11/79	731.74	18.23 (7015)
G–111	25/03/66	904.01	13.43 (7926)
G–115	16/05/78	212.04	80.70 (4068)
G–125	24/10/75	1016.92	16.23(10258)
G–131	15/05/78	320.29	84.42 (4380)
G–142	14/03/67	288.27	62.47 (4283)
G–157	10/08/67	1029.64	45.16 (8428)
G–220	14/11/79	1011.06	15.36 (8035)
G–268	15/05/78	902.37	12.61 (8493)
G–269	11/10/78	364.85	28.81 (5924)

The quantities in parentheses denote the period of the periodic term in counter units

Model used is given by equation (5.7) where LINEAR implies n=1 and k=0; PERIODIC implies n=1 and k=1.

Table 7 — Summary of the results of modelling the Calibration Table 1 counter readings from 2000 to 4000 using various order polynomials and a linear function plus a periodic term.

Gravity Meter	Calibration Date	Root Mean Square Fit in μgal Linear	Second	Third	Periodic
G-10	25/10/60	352.81	27.41	23.98	7.64 (4631)
G-44	25/04/63	209.56	11.69	11.59	3.69 (5104)
G-68	23/03/64	181.08	16.36	14.23	13.60(10696)
G-81	07/08/64	293.63	14.19	9.85	9.83(26155)
G-103	07/09/65	198.91	13.48	9.21	5.17 (5920)
G-103	14/11/79	214.81	13.43	3.90	0.99 (8647)
G-111	25/03/66	248.94	16.24	4.18	3.30(11264)
G-115	16/05/78	84.28	6.26	5.39	1.61 (4756)
G-125	24/10/75	243.53	14.30	9.04	8.54(10349)
G-131	15/05/78	125.81	14.28	4.68	0.75 (6049)
G-142	14/03/67	128.35	21.36	9.96	6.03 (4721)
G-157	10/08/67	229.13	14.34	6.73	7.32 (8248)
G-220	14/11/79	266.25	9.99	5.05	2.65 (9014)
G-268	15/05/78	248.60	30.49	6.12	2.11 (7551)
G-269	11/10/78	121.34	14.23	4.69	2.62 (6442)

The quantities in parentheses denote the period of the periodic term in counter units.

Model used is given by equation (5.7) where LINEAR implies n=1 and k=0; SECOND implies n=2 and k=0; THIRD implies n=3 and k=0 and PERIODIC implies n=1 and k=1.

3rd order polynomial and the sinusoid model was relatively small. However, there was, generally, a noticeable improvement in the RMS fit as the degree of the polynomial increased.

It is known that the Calibration Table 1 for each gravity meter was produced by an arbitrary method of curve fitting of the relative scale factors from the factory calibration procedure. Also, in general, the least count of the counter reading recorded is equivalent to approximately 10 μgal. From this information, it would appear that either a sinusoid or 3rd order polynomial model would be a good analytic representation of the Calibration Table 1 information in the range of the counter readings observed along the United States Mid-Continent Calibration Line. The question comes up as to which model should be used.

5.2.2 Sinusoidal Model

The reason a sinusoidal model was selected over a 3rd order polynomial model is based on the fact that, after the linear trend is removed, the sinusoid represents a simple curve where as the 3rd order polynomial is a composite of two simple curves, one a function of x^2 and the other a function of x^3. This makes the polynomial model somewhat undesirable because changes in the coefficients of the 2nd and 3rd order terms are difficult to relate to changes in the basic characteristic behavior of the curve it is representing. The sinusoidal model is easier to interpret what changes in the value of the amplitude, period or phase angle represent. By using the

sinusoidal model, the characteristic behavior of the Calibration Table 1 can be classified by the period of the sinusoid which best represents the information.

The determination of the coefficients of the sinusoidal model given by equation (5.7) where n = 1 and k = 1 was done by a least squares adjustment of the set of non-linear observation equations. For the adjustment, the counter readings were assumed to be without error and the variance associated with each independent and uncorrelated observation (value in milligal) was assumed to be of the same constant value. The solution was iterated until the change in V^tPV was less that 1 part in 10 billion. To assure that the adjusted value of the period of the sinusoid would represent a long wave length, the period was weight constrained to permit a proper solution to be achieved. The actual value of the weight needed depended on the initial estimates of the coefficients. The determination of the weights and the good estimates for the coefficients involved a lot of trial and error. It was not uncommon to perform 150-300 iterations before a solution was achieved based on the desired accuracy of V^tPV. The large number of iterations required indicated there was a very strong dependency that exists between the period and phase angle in the sinusoidal model. With the period and phase angle being so highly correlated, it is not feasible to include both quantities as parameters in an adjustment model and expect it to converge rapidly. Therefore, either the period or phase angle must be fixed or heavily constrained within an adjustment.

When the period is heavily constrained which essentially fix its value, a solution can easily be achieved in a few iterations. By obtaining solutions for a number of different periods, graphs similar to Figure 20 or Figure 21 can be produced for each gravity meter which show the approximate relationship between values of the coefficents of the sinusoidal model and the value of the RMS of the residuals. As can be seen in either Figure 20 or Figure 21, changes in the value of period near the solution with the minimum RMS of the residuals results in little change in the RMS of the residuals but means large changes in the phase angle, amplitude and linear scale factor terms. Using graphs, similar to Figure 21, estimates for the value of the sinusoidal model coefficients can be determined for different period of the sinusoid. From these graphs, a period near the minimum RMS of the residuals was selected for each gravity meter. Since the value of the period has little effect on the RMS of the residuals, all periods used in the final adjustment were rounded off to the nearest thousand of counter units. To see if the rounding off would have any effect on the final adjustments, a number of preliminary adjustments were done for various values of the periods near the rounded off value. The results indicated That the least squares soultion for the model used would not be effected by using the rounded off periods.

5.3 Mathematical Models for Gravity Meters

The basic equation used for the 'G' gravity meter is of the form given by equation (2.1) where C includes such things as the correction

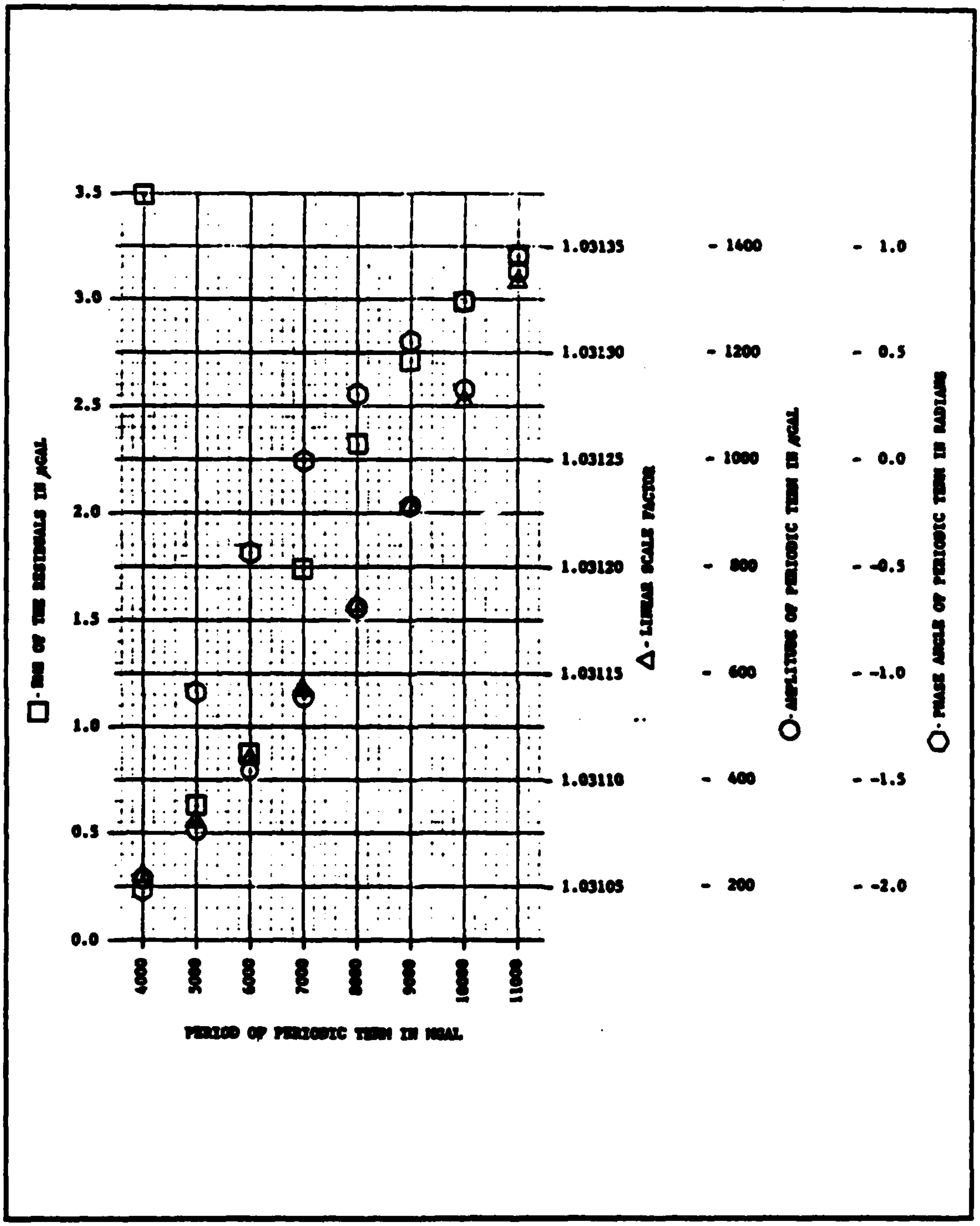


Figure 20 — The results of a number of least squares fits of a linear function plus a periodic term to the data from the Calibration Table 1 dated 17 October 1977 for gravity meter G-81 for selected values of the period.

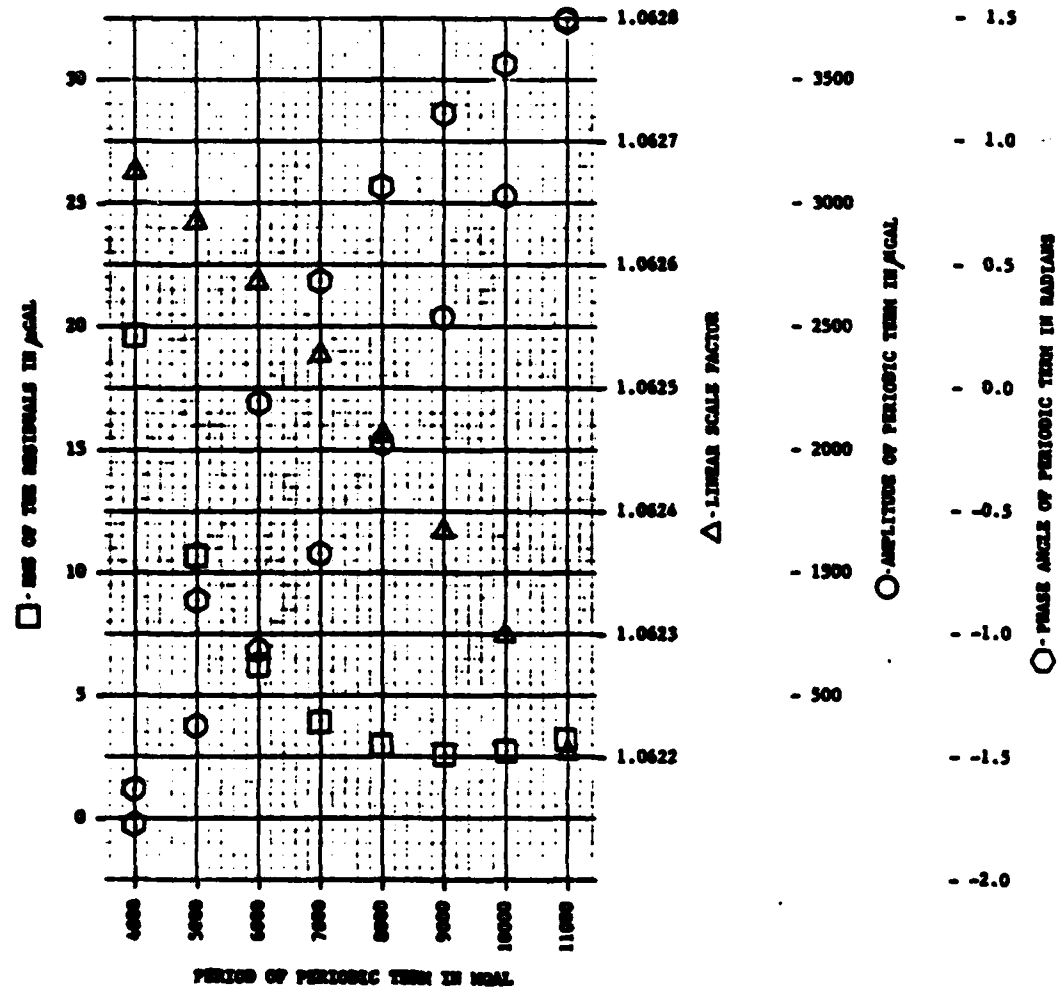


Figure 21 — The results of a number of least squares fits of a linear function plus a periodic term to the data from the Calibration Table 1 dated 14 November 1979 for gravity meter G-220 for selected values of the period.

for earth tides, the correction for height of instrument above the station, the correction for a possible time dependent drift of the instrument and it could include the correction for other environmental or geophysical effects such as changes in the ground water level and changes in the atmosphere above the station.

In order to make the gravity meter observations independent of the epoch of the observation, any time dependent effects must be included in C. One time dependent effect that must be removed is caused by gravitational attraction of the sun and the moon, commonly referred to as the earth tide.

5.3.1 Earth Tide

The determination of the earth tide is accomplished by computing the effects of the sun and the moon on the gravity value of the station at the epoch of the observation.

The program used to compute the earth tide was obtained from the Finnish Geodetic Institute. The program computes the vertical component of the tidal force to an accuracy of 0.01 μgal based on the assumption of a completely rigid homogeneous ellipsoid of revolution [Heikkinen, 1978]. Since the earth is not really a completely rigid homogeneous ellipsoid of revolution, in order to obtain a realistic value for the vertical component of gravity, the tidal force obtained is scaled. The value of this scale factor depends on regional conditions, but a world-wide conventional value of 1.16 was recommended by Resolution N 2 of the International Association of Geodesy in

Canberra, Australia, 1980 [IAG, 1980]. In the computation of the earth tide for each observation, a scale factor of 1.16 was applied to the computed vertical component.

The implementation of the Finnish earth tide program as received and given in Heikkinen [1978] proved to be very inefficient as far as input/output (I/O) was concerned. By making some simple I/O changes which basically involved modifying the subroutine GETREC so that the I/O was done unformatted and only once during the execution of the program, the computation time required to compute the earth tide was reduced by a factor of approximately 10. After the modifications were made, it took approximately 0.3 seconds per earth tide correction on an Amdhal 470 V/6-II computer operated by the Instructional and Research Computer Center (IRCC) at The Ohio State University.

One other minor problem involving the computer program was noticed and corrected in the version of the program used in this study which should be mentioned. The program requires information about the difference between ephemeris and universal time but the program only knows the differences for the years 1973, 1975-1979. If earth tide corrections were to be computed for observations made during any other year, it would require appropriate addition of coding in the MAIN program. Since the current difference is almost one minute, by not including this difference could result in a systematic error of 0.5 μgal [Heikkinen, 1978]. Since the time for each observation used was recorded to the nearest minute, it can be expected that the accuracy of the theoretical earth tide computed would be on the order

of 0.5 μgal.

5.3.2 Correction for the Height of Instrument

In order to assure that the observations at a site refer to the same point, any correction for the height of instrument must be made. The correction for the height of instrument above the station is made by assuming a local gradient of gravity for the station. Since this information is not generally available, the value of normal gravity, −3086 eötvös (10^{-1} μgal/m), is used. Thus the correction to be made is equal height of instrument in meters times −308.6 which gives the correction needed in units of μgal. This correction could be the source error if there were a large variation in the height of an instrument at a station because the local gravity gradient can easily differ from the normal gravity gradient by 20 to 30 per cent or more [Marson and Alasia, 1980]. This error could be on the order of 1 μgal per cm of elevation difference.

5.3.3 Instrumental Drift

In order to check for the possibility of drift in the 'G' gravity meter, a model must be assumed which will represent this behavior. Since the drift is believed to be the accumulative effect of a number of small tares [Burris, 1980, private communication], the hypothesis that these tares occur uniformly could be adopted which could be represented by a simple linear model such as

$$k(t - T_o) \tag{5.8}$$

where

k — the drift rate of the instrument,

t — the epoch of the observation,

T_o — some initial epoch.

However, the appropriateness of including such a term is questionable because of the seemly unpredictable and erratic behavior of the tares. Small tares can occur whenever there is any change in the operating condition of the gravity meter, such as would be caused by vibrations during transportation, changes in the ambient temperature which might causes the heater in the instrument to cycle on and off repeatedly, or changes in the atmospheric pressure.

A number of least squares adjustments were performed using observations made along the United States Mid-Continent Calibration Line and a linear model similar to that expressed in equation (5.2) which incorporated a drift term for each gravity meter. A number of adjustments were made involving different combinations of the various types of drift rate terms. The drift terms tried involved solving for a linear drift rate for each instrument, a linear drift rate for each trip, and even a linear drift rate involving only ties between the same stations. The latter drift rate term could be referred to as night drift because the ties were made between the same stations which, generally, occurred as a result of overnight stops.

For example, when a linear drift rate for each instrument was included in the model, there was no evidence of any linear drift rate for any instrument which had observations made over a period of a month or more. Observations for instruments which involved a more limited period did indicate a possible linear drift, but these drifts were discarded because there were too few observations or the time period of the observations was too short to draw the conclusion that the drift was the behavior of the gravity meter. The adjustments performed indicated that the linear drift rates of the type mentioned were not representative of the characterisitics of any of the gravity meters used in this study.

This fact became more apparent when linear drift rates for each trip were included in the model. The linear drift rates determined for a gravity meter were inconsistent both in magnitude and direction. This would lead one to conclude that if there were a linear drift rate for a gravity meter, it is not of an instrumental nature but possibly a function of the instruments' handling, mode of transportation or some other unknown cause.

No conclusion could be drawn concerning the existence of a linear night drift rate due to the lack of a sufficent number of 'night' observations for any gravity meter.

It is worth mentioning that although the gravity meter observations used in this study indicated no predictable linear drift rate behavior, this does not mean that a linear drift rate for the gravity meters does not exist. It is quite possible that if all the observations were made

so the ties formed ladder loops instead of the modified ladder and line loops which typifies the observations in this study, the conclusion might be different.

The apparent erratic behavior of the linear drift rate is demonstrated latter on in section 6.3.

To demostrate the apparent erratic behavior of a possible linear drift, an adjustment was performed using the same model as used in ADJUSTMENT B, see section 6.3, with T1 and T2 both having the value of 24 hours. The residuals obtained were then plotted against time. Part of the plot for gravity meter 'G—81' can be seen in Figure 22. As mentioned, Figure 22 shows no indication of any uniform linear instrumental drift rate.

5. .4 Mathematical Models Using Value in Milligal

The values in milligal must be used as the observables in an adjustment whenever it is unreasonable to use counter readings as the observables. This occurs when a gravity meter has been used over only a limited range of gravity values, generally less than 600 mgal. In order to use the counter readings as the observables requires solving for a long wave length sinusoidal term whose period is always larger than 4000 counter units which is equivalent to approximately 4000 mgal. Trying to solve for the coefficients of the sinusoidal term using a limited range of data would lead to poorly determined values of the coefficients which could be reflected in incorrect values for other parameters of the adjustment, such as, linear scale factor terms and

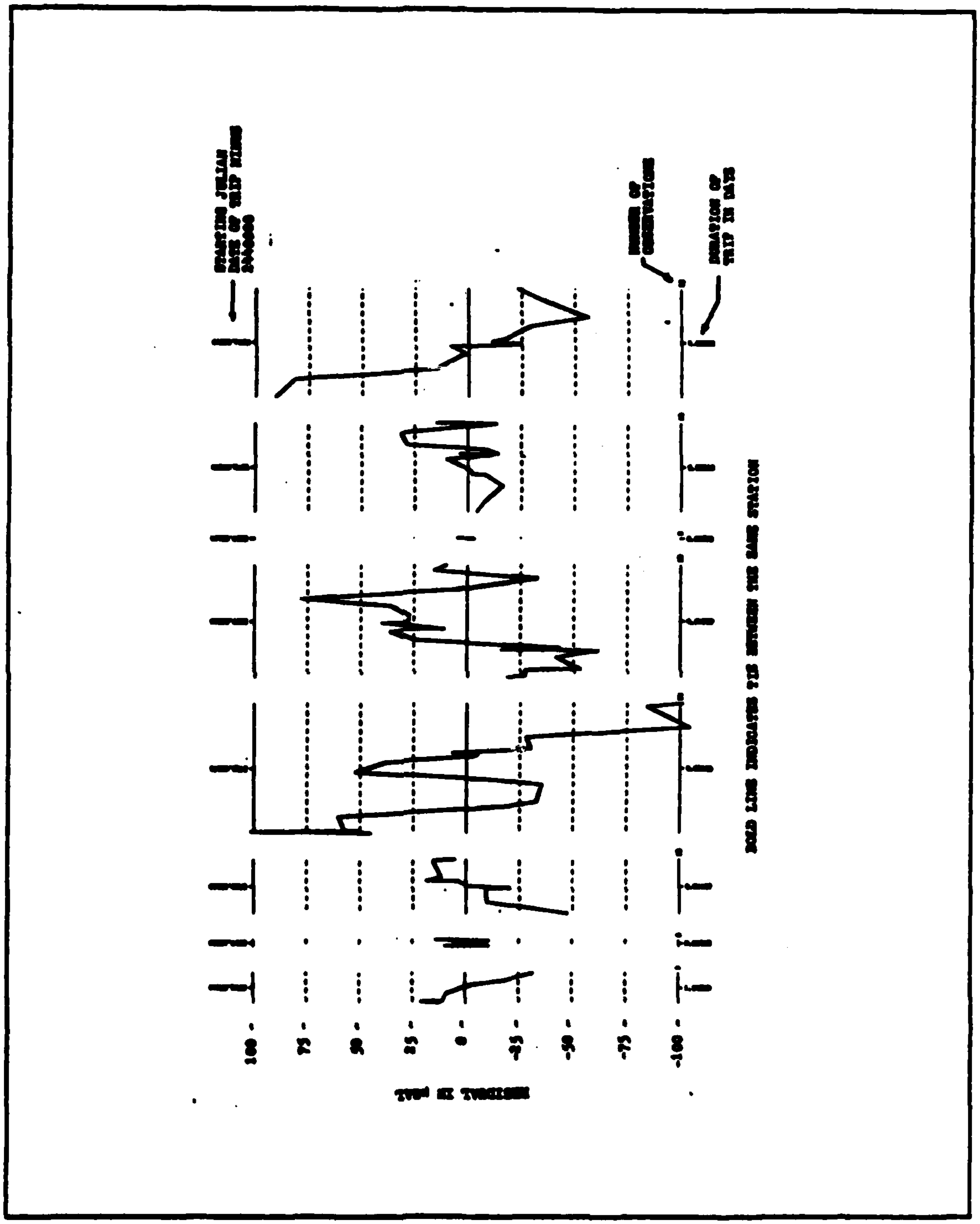


Figure 22 — Partial plot of the residuals for gravity meter G-81 after ADJUSTMENT B which did not include any linear drift rate term.

gravity station values. Therefore, whenever a gravity meter is used which has observations spanning a limited gravity range, the appropriate model to be used would be one using the value in milligal as the observable.

The actual observation made with the gravity meter is the reading made of the position of counter and the dial after the instrument has been nulled. The reading is generally recorded in counter units. The readings in counter units are to be related to a value in milligal through the use of the Calibration Table 1. If one believes that the gravity meter's behavior remains constant with time and is reflected via the Calibration Table 1, then the value in milligal obtained from the Calibration Table 1 would be correct except for a possible correction to the initial offset and to a scale factor. These correction terms could be considered as part of a required transformation that must be applied when using the Calibration Table 1. For a simple transformation, one could consider a linear transformation which would imply

$$z = ly + R \tag{5.9}$$

where

- R – some offset value to be applied to Calibration Table 1,
- l – scale factor required to be applied to Calibration Table 1,
- y – value in milligal from Calibration Table 1.

Then by rewriting equation (2.1) where $f(O) = z$, $C = c_1 + c_2 + c_3$ and $z = ly + R$ results in

$$z + C + S - G = 0 \tag{5.10}$$

where

c_1 – earth tide correction,

c_2 – height of instrument correction,

c_3 – other corrections due to environmental factors.

If one assumes that station G_i is observed at time i and station G_j is observed at time j, then by differencing the two consecutive equations of the form (5.10) for the two different stations, one obtains

$$l(y_i - y_j) + C_i - C_j - G_i + G_j = 0 \tag{5.11}$$

Since it is assumed that the Calibration Table 1 supplied is relatively good, the value of the scale factor l will be very close to one, and we can rewrite l as l + l' giving

$$y_i - y_j + l'(y_i - y_j) + C_i - C_j - G_i + G_j = 0 \tag{5.12}$$

The similar transformation needed for the Calibration Table 1 could be expressed as a higher order polynomial transformation giving

$$y_i - y_j + \sum_{k=1}^{n} l'_k(y_i^k - y_j^k) + C_i - C_j - G_i + G_j = 0 \tag{5.13}$$

where

n – the order of the transformation,

l'_i – the scale factor corresponding to the i–th degree,

Then if two successive observations were differenced, letting $(y_i - y_j)$ be equal to y_{ij}, the resulting equation would be the basic model for gravity differences. It is apparent that if $n > 1$, gravity differences can no longer be used as the observable in the model given by equation (5.13) since $(y_i^n - y_j^n) = (y_i - y_j)^n$ only when $n = 1$. For example, if $n = 2$, the right hand side of the previous equation becomes $y_i^2 - 2y_iy_j + y_j^2$ which is not equal to $y_i^2 - y_j^2$ except in the trivial case when $y_i = y_j$.

5.3.5 Mathematical Model for Counter Reading

The functional relationship of the observable, f(O), as given in equation (2.1) can be expressed approximately by equation (5.7) with $n = 1$ and $k = 1$. Using the information in equation (5.7), equation (2.1) can be rewritten as

$$\sum_{i=1}^{n} l_i d^i + \sum_{j=1}^{k} A_j \cos\left(\frac{2\pi d}{T_j} + \omega_j \right) + S + C - G = 0 \tag{5.14}$$

and similarly equation (2.2) can be rewritten as

$$\sum_{i=1}^{n} l_i (d_1^i - d_2^i) + \sum_{j=1}^{k} \sum_{m=1}^{2} (-1)^{m+1} A_j \cos\left(\frac{2\pi d_m}{T_j} + \omega_j \right) + C_1 - C_2 - G_1 + G_2 = 0 \tag{5.15}$$

Equation (5.15) then becomes the basic mathematical model for the 'G' gravity meter when the observed counter readings are used as the observables.

The use of equation (5.15) implies there must be a sufficient distribution of the observations made with a gravity meter to solve for the coefficients of the sinusoid. What actually constitutes a sufficient distribution is difficult to state. But clearly, certain distributions of the observations would not be desired. For example, if all the observations were made at stations having approximately the same gravity value, then the model expressed by equation (5.15) would be inappropriate.

CHAPTER SIX

TESTING OF THE MATHEMATICAL MODELS

6.1 Solution of the Mathematical Model

The mathematical model used for the gravity adjustment program can be expressed by $F(X,L) = 0$, where F is a vector of nonlinear functions of the parameters X and the observables L. The problem is to find the vectors X and L such that $V^t PV$ is a minimum subject to the constraint, $F(X,L) = 0$. Even though the weight matrix for the observations, P, is not a unit matrix, the method used to solve this problem is commonly referred to as "least squares" even though the more appropriate name would be "weighted least squares".

The solution of this type of nonlinear least squares problem is by a Newton-Gauss iteration, which expresses the nonlinear function F as a Taylor's series about some initial values of the parameters and observables. By ignoring the second and higher order terms of the Taylor's series, it reduces to solving a linear least squares problem. If the function F is not satisfied by the adjusted parameters and observables obtained, the solution is iterated by using the adjusted parameters and observables as the new points of expansion for the Taylor's series [Pope, 1972].

The multivariate Taylor's series expansion of $F(X,L)$ about X_o, L_o with the condition $F(X,L) = 0$ becomes

$$\left.\frac{\partial F}{\partial X}\right|_{X_o,L_o} (\overline{X} - X) + \left.\frac{\partial F}{\partial L}\right|_{X_o,L_o} (\overline{L} - L_o) + F(X_o,L_o) = 0 \tag{6.1}$$

with the notation meaning the partials are to be evaluated at X_o, L_o. To simplify the notation, let $X = \overline{X} - X_o$, $A = \partial F/\partial \overline{X} \big|_{X_o,L_o}$ and $B = \partial F/\partial \overline{L} \big|_{X_o,L_o}$. The residuals, V, of the observables are given by $V = \overline{L} - L_b$ where $\overline{L}$ is the adjusted value of the observable and L_b is the observed value. This relation can be expressed as

$$\overline{L} - L_o = V + L_b - L_o \tag{6.2}$$

which allows equation (6.1) to be written as

$$\left.\frac{\partial F}{\partial X}\right|_{X_o,L_o} (\overline{X} - X_o) + \left.\frac{\partial F}{\partial \overline{L}}\right|_{X_o,L_o} V + \left.\frac{\partial F}{\partial \overline{L}}\right|_{X_o,L_o} (L_b - L_o) + F(X_o,L_o) = 0 \tag{6.3}$$

Using the notation previously mentioned yields

$$AX + BV + B(L_b - L_o) + F(X_o,L_o) = 0 \tag{6.4}$$

and by letting $W = F(X_o,L_o) + B(L_b - L_o)$, equation (6.4) becomes

$$AX + BV + W = 0 \tag{6.5}$$

With $\sum_{L_b}$ being the covariance matrix of the observables and σ_o^2 being some constant, the weight matrix, P, for the observables is related to

$\sum_{L_b}$ by

$$P = \sigma_o^2 \sum_{L_b}^{-1} \tag{6.6}$$

The derivation of the standard linear least squares problem is well documented [Hamilton, 1964; Mikhail and Ackermann, 1976] and yields

$$\hat{X} = -(A^t(BP^{-1}B^t)^{-1}A)^{-1}A^t(BP^{-1}B^t)^{-1}W \tag{6.7}$$

$$\hat{V} = -\ P^{-1}B^t(BP^{-1}B^t)^{-1}(AX + W) \tag{6.8}$$

with the best estimate of the vectors X and L being given by

$$\overline{X}_o = X_o + \hat{X} \tag{6.9}$$

$$\overline{L}_o = L_b + \hat{V} \tag{6.10}$$

If these best estimates of the parameters and observables do not satisfy the function F, the solution needs to be iterated with the point of expansion of the Taylor's series now being $(\overline{X}_o,\overline{L}_o)$. This means, the partials are now evaluated at the new point of expansion and the W matrix is recomputed. If $\hat{V}$ denotes the vector of residuals for the observables from the previous iteration, then W can be expressed as

$$W = F(X_o,L_o) - B\hat{V} \tag{6.11}$$

noting that for the zeroth iteration $\hat{V}$ will be a zero vector.

The iterative process continues until the mathematical model is satisfied. Due to round-off error in the computer's representation of

values and computation of the matrix operations, the chances are the model will never truly be satisfied. However, it can generally be satisfied so $F(X,L)<\varepsilon$, where ε is a vector as small as desired within the limits of the round-off error of the computations. Therefore, the iteration should continue until $F(X,L)<\varepsilon$ is satisfied. The number of iterations required will depend on how close the initial parameter values and observables are to the values required to satisfy $F(X,L) = 0$.

The mathematical model $F(X,L) = 0$ is really the composite of two functions, $F_1(X,L_1) = 0$ and $F_2(X,L_2) = 0$, where the vector of parameters, X, is the same for F_1 and F_2. Further, L_1 is the vector of observables from gravity meters and L_2 is a vector of derived absolute gravity station values from either permanent or portable absolute gravity measuring devices. This implies that there are "observed" gravity station values. These values are "observed" in the sense that a variance for the station's gravity value exists. This type of model has been referred to as the "combined observation and parameter model with weighted parameters" [Uotila, 1967].

If one partitions the A, B, P and W matrices into two parts; part 1 for $F_1(X,L_1) = 0$ and part 2 for $F_2(X,L_2) = 0$, and letting $M = BP^{-1}B^t$ where the observed values of L_1 and L_2 are not correlated then

$$M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} P_1^{-1} & 0 \\ 0 & P_1^{-1} \end{bmatrix} \begin{bmatrix} B_1^t & 0 \\ 0 & B_2^t \end{bmatrix} \tag{6.12}$$

It can then be shown that equations (6.7), (6.8) and (6.10) can be expressed as

$$X = -\left\{ \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}^t \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}^{-1} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \right\}^{-1} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}^t \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}^{-1} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \tag{6.13}$$

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = - \begin{bmatrix} P_1^{-1} & 0 \\ 0 & P_1^{-1} \end{bmatrix} \begin{bmatrix} B_1^t & 0 \\ 0 & B_2^t \end{bmatrix} \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} X + \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \right\} \tag{6.14}$$

$$\overline{L}_o = \begin{bmatrix} \overline{L}_{o_1} \\ \overline{L}_{o_2} \end{bmatrix} = \begin{bmatrix} L_{b_1} \\ L_{b_2} \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{6.15}$$

which can be further simplified to

$$X = -(A_1^t M_1^{-1} A_1 + A_2^t M_2^{-1} A_2)^{-1} (A_1^t M_1^{-1} W_1 + A_2^t M_2^{-1} W_2) \tag{6.16}$$

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = - \begin{bmatrix} P_1^{-1} B_1^t M_1^{-1} (A_1 X + W_1) \\ P_2^{-1} B_2^t M_2^{-1} (A_2 X + W_2) \end{bmatrix} \tag{6.17}$$

$$\overline{L}_o = \begin{bmatrix} \overline{L}_{o_1} \\ \overline{L}_{o_2} \end{bmatrix} = \begin{bmatrix} L_{b_1} + V_1 \\ L_{b_2} + V_2 \end{bmatrix} \tag{6.18}$$

In addition, it can be shown that $\sigma_o^2 = V^t PV/D.F.$ where D.F. represents

the degrees of freedom of the adjustment which is equal to the number of equations minus the number of parameters in the model and

$$V^tPV = (AX + W)^tM^{-1}W \tag{6.19}$$

or

$$V^tPV = V_1^tP_1V_1 + V_2^tP_2V_2 = (X^tA_1^t + W_1)M_1^{-1}W_1 + (X^tA_2^t + W_2)M_2^{-1}W_2 \tag{6.20}$$

Additional examination of the structure of the A, B, P and W matrices reveals that A_1, B_1, P_1 and W_1 matrices can be further partitioned by trips and A_2, B_2, P_2 and W_2 matrices can be partitioned by each set of absolute gravity observation equations. Applying this information, equations (6.16), (6.17) and (6.20) can be written as

$$X = -\left\{\sum_{i=1}^{2}\ \sum_{j=1}^{r_i} A_{ij}^tM_{ij}^{-1}A_{ij}\right\}^{-1}\left\{\sum_{i=1}^{2}\ \sum_{j=1}^{r_i} A_{ij}^tM_{ij}^{-1}W_{ij}\right\} \tag{6.21}$$

$$V = -\begin{bmatrix} P_{11}^{-1}B_{11}^tM_{11}^{-1}(A_{11}X + W_{11}) \\ \vdots \\ P_{1s}^{-1}B_{1s}^tM_{1s}^{-1}(A_{1s}X + W_{1s}) \\ P_{21}^{-1}B_{21}^tM_{21}^{-1}(A_{21}X + W_{21}) \\ \vdots \\ P_{2t}^{-1}B_{2t}^tM_{2t}^{-1}(A_{2t}X + W_{2t}) \end{bmatrix} \tag{6.22}$$

$$V^tPV = \sum_{i=1}^{2}\ \sum_{j=1}^{r_i}(X^tA_{ij}^t + W_{ij})M_{ij}^{-1}W_{ij} \tag{6.23}$$

where $s \equiv r_1$ and $t \equiv r_2$ with r_1 being the number of gravity meter trips made and r_2 being the number of sets of absolute gravity observation

equations.

The B_l and P_l matrices have a very well defined pattern that can be exploited in forming the M_l matrices. Assuming that all observations made with a particular gravity meter over a trip have the same variance, σ_l^2, where l denotes the trip number, then the covariance matrix of the gravity meter observation for trip l is given by

$$P_{ll}^{-1} = \sigma_l^2 \, I \tag{6.24}$$

where I is and nxn identity matrix and n is the number of observations in the trip.

The elements of the B matrix for the gravity meter observations for trip l are given by

$$\begin{aligned}
b_{ii} &= \frac{\partial F}{\partial l_i} && 1<i<n-1 \\
b_{ij} &= \frac{\partial F}{\partial l_i} && 1<i<n-1 \quad \text{and} \quad j=i+1 \\
b_{ij} &= 0 && j>i=1 \quad \text{and} \quad j<i
\end{aligned} \tag{6.25}$$

where l_j is the j-th observation in trip l. The patterned structure of the B matrices for the gravity meter observations is similiar to

$$= \begin{bmatrix} * & * & 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & * & * & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & * & * & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \ldots & * & 0 \\ 0 & 0 & 0 & 0 & 0 & \ldots & * & * \end{bmatrix} \tag{6.26}$$

where * represents a non-zero element and 0 represents a zero element

of the matrix B.

Since

$$M_{11} = B_{11}P_{11}^{-1}B_{11}^{t} \tag{6.27}$$

by substituting equation (6.24) and (6.25) into (6.27) gives

$$\frac{1}{\sigma_1^2} M_{11} = B_{11}B_{11}^{t} \tag{6.28}$$

with the value of the elements of M_{11} given by

$$\begin{aligned} m_{ii} &= b_{ii}^2 + b_{i+1,i+1}^2 && 1<i<n-1 \\ m_{ij} &= m_{ji} = -b_{ij}^2 && j=i+1 \quad \text{and} \quad 1<i<n-1 \\ m_{ij} &= m_{ji} = 0 && j>i+1 \quad \text{and} \quad 1<i<n-1 \end{aligned} \tag{6.29}$$

There are times when the B matrix has its elements satisfying

$$b_{ii} = -b_{ij} \qquad j=i+1 \quad \text{and} \quad 1<i<n-1 \tag{6.30}$$

This occurs during the initial adjustment iteration when the parameters other than the gravity station values have an initial value of zero. In which case the inverse of M_{11} matrix can be obtained directly [Gergory and Karney, 1969, pp 45–46] from the expression

$$\sigma_1^2 M_{11}^{-1} = \frac{1}{n+1} C \tag{6.31}$$

where n is the order of the matrix M_{11}, k is the value of a non-zero element of the B matrix squared and the elements of C are given by

$$C_{ij} = C_{ji} = i(n - j + 1) \qquad i \le j \tag{6.32}$$

6.2 Computer Program Algorithms

The computer program developed to handle the adjustment was written in PL/1 and designed to run on the Amdahl 470 V/6-II computer using a virtual core region of 2048 Kbytes. The reason the program was written in PL/1 was to allow for dynamic allocation of arrays and the ease of handling output formatting.

The PL/1 program written to perform the gravity network adjustment was designed to meet the following requirements:

a) the program should be able to handle any number of unknowns less than 32767.

b) the program should be able to determine what stations are in the network, what instruments are being used and should be able to break the observation set into trips with the capability of handling instrumental tares.

c) the program should be able to use different models for each instrument.

d) the program should allow for full variance-covariance weighting of any parameter.

e) the program should be able to do post adjustment analysis.

f) the program should be able to iterate on the solution.

g) the program should have as few fixed values and limits as

possible.

For the most part, all the above requirements were satisfied except for one minor restriction. Due to a PL/1 limitation which restricts the size of an array subscript to a maximum of 32767, the present limit on the number of observations allowed in any trip is 181 observations. This restriction is necessary because when the weight matrix, M^{-1}, for the equations of a trip is full and it is required that all elements of the matrix to be in core at the same time. This limitation has not caused any problems in this study.

The program can be thought of as consisting of three sections: Pre-Processor, Adjustment and Post-Analysis. The options for each section are controlled by values assigned to a fixed binary external array of 130 elements.

6.2.1 Pre-Processor Section

By prior analysis, the input data set of the observations had all detectable blunders removed and any large suspected tares flagged. The method used to detect the tares and blunders involved determining the mean gravity difference between all of the stations which were tied. The gravity difference for each gravity meter tie was determined using the the values in milligal from the gravity meter's Calibration Table 1 which had been interpolated from their counter reading observations. Any large difference which was greater than 100 μgal from the mean gravity difference between the two stations tied was flagged so the observations involved could be investigated. In the majority of the

cases, the flagged observations had notations on the original observation sheets which explained why a large difference might has been encountered. The most common notations were "beam vibrations" and "meter off-heat". Using this information, it was possible to determine which observations should be treated as a tare and which ones were blunders. A blunder is indicated when two consecutive gravity meter ties are flagged as suspect. The common observation involved in the two ties would be labelled as a blunder and removed from the data set.

The purpose of this section is to determine the make-up of the input data set of observations. It determines how many different stations and instruments there are and breaks the observations into trips based on a number of criteria. If it has been determined that a tare has occurred between observations, then the current trip ends with the last observation before the tare and a new trip begins with the first observation after the tare. Based on the time interval between observations, a new trip is begun if the time interval exceeds a maximum time specified which depends on whether the time interval is between the same station or different stations. After a tentative trip has been identified, it is checked to see that there are at least a minimum number of observations in the trip. If not, then the observations in that trip are deleted and are not included in the adjustment.

After the observations to be used have been identified and broken into trips, the stations are assigned parameter numbers based on ascending order of their assigned identification code. The

identification code is either the IGB (International Gravity Bureau) number, if the station was part of the IGSN 71, or some other arbitrary assigned code for any other station. The instruments are assigned a parameter number based on ascending order of their assigned data set number and instrument number.

Any instrument requires a model different from the default model, which was the model using the value in milligal as the observable was code, input, saved, and the remaining model parameters were assigned. This is followed by the assignment of initial values to all the parameters based on input information. A final check is made to be sure that there are sufficient redundancies for each instrument and that all the stations are inter-connected by observational ties to guard against trying to solve a singular system.

6.2.2 Adjustment Section

The purpose of this section is to actually perform the weighted least squares adjustment. For each iteration performed, all the non-zero partials with respect to the parameters and observables are computed along with the model misclosures and the full weight matrix, M^{-1}, for each trip is formed. See Appendix A for a detailed discussion of how this is accomplished. The contributio.ı to the normal equations for each trip is then computed by performing only non-zero multiplication. See Appendix B for a detailed discussion of how this is done.

The solution is iterated until the model is satisfied or the iteration count exceeds the maximum allowed. The model F(X,L) is satisfied for some predetermined value of ε, when the condition $F(X,L) < \varepsilon$ is fulfilled.

6.2.3 Post-Analysis Section

The purpose of this section is to do a post-adjustment analysis of the results of the adjustment. In this section the variance of each residual is computed, and the normalized residuals are formed. For details on how this is done, see Appendix C. Any large normalized residual is listed to be checked for possible blunder or tare. The meaning of large is based on the Tau-criteria as outlined by Pope [1974] and depends on the number of observations and the significance level selected. In this section, the root mean square of the residuals for each instrument is computed. The program also computes an estimate of the aposteriori accuarcy of the observable by trip and instrument based on the estimated initial variance of the observable, its residual, and the aposteriori variance of the residuals.

6.3 Testing Various Models

For the comparison of the various possible models, three different adjustment were performed using the same set of observation data. All adjustments were made using the same set of absolute sites as control. The estimated stand ard errors assigned to the absolute sites are given in Table 8. The values in Table 8 were based on the assumption that the

value of gravity determined at a site using the absolute gravity meter developed by either Marson and Alasia or Hammond had the same accuracy and gravity values did not include any bad determinations. By reviewing the gravity stations in Table 4 and Table 5, see Section 4.2, which had multiple determinations, it can be seen that 156E05, Great Falls, values differ by 101 μgal and that 15211A, Boston, values differ by 109 μgal. It follows that with a probability of 0.99, a single determination made with either apparatus would lie within 2.57σ of the mean gravity value for that station. If it were assumed that the mean gravity value at a station were located at the middle of the two extreme gravity values, then the accuracy of a determination could be estimated to be better than $s = ((x2 - x1)/2)/2.57$ where x2 is the largest gravity value determined, x1 is the smallest gravity value determined, and s is the estimate of the accuracy of a determination. Using this relationship, the accuracy of the absolute gravity meter determinations which could account for the different gravity values reported would be approximately 20 μgal instead of the reported 10 μgal accuracies. The accuracy of 115V01, Casper, was kept at 25 μgal because there was good no reason to lower its reported accuracy.

The variance for each gravity meter observation was assumed to be the same for all gravity meters. The variance assigned to the observations made with the gravity meters were 0.0004 counter units^2 or 400 μgal^2, depending on which type of observable was used in the model for each gravity meter. The above values were based on preliminary adjustments which indicated that no large differences existed between

the aposteriori estimates of the accuracy of the observations for any gravity meter. A total of 4532 gravity observations were considered of which 48 was rejected because of detected tares or blunders. The remaining 4484 observations, made with 27 different gravity meters, resulted in 837 different trips based on the value of 24 hours for T1 and 6 hours for T2 (see Section 3.3.2). The minimum number of observations in any trip is 2 while the largest trip consisted of 23 observations. See Figure 23 for a histogram of the frequency of the trips.

The three adjustments will be referred to as ADJUSTMENT A, ADJUSTMENT B, and ADJUSTMENT C. All adjustment models included a linear scale factor term but no drift terms for each gravity meters. ADJUSTMENT A was performed using as observables the observed counter units transformed to their values in milligal by using the factory supplied Calibration Table 1. ADJUSTMENT B and C were performed using as the observable the observed counter readings whenever the range and distribution of the data justified their use. Otherwise, the values in milligal were used. Both ADJUSTMENT B and C models included a long wave sinusoidal term to represent the Calibration Table 1 information. ADJUSTMENT B's model contained no periodic screw error terms whereas ADJUSTMENT C's model included periodic screw error terms having a period of 1286/17 counter units for selected gravity meters. All gravity meters used in the adjustments had "old" gear boxes installed in them. The gravity meters which included a periodic screw error term were selected based on the number and distribution of their

Table 8 — List of absolute stations used in ADJUSTMENTS A, B, and C.

IBG/GSS Code	Station Name	Gravity Value in μgal	Estimated Standard Error in μgal
119S04	McDonald Obs.,TX	978820097	20
119S03	McDonald Obs.,TX	978828655	20
08150C	Miami,FL	979004319	20
11926A	Holloman,NM	979139592	20
119C03	Mt. Evans,CO	979256059	20
119C01	Trinidad,CO	979330370	20
11994H	Denver,CO	979598272	20
15505D	Boulder,CO	979608498	20
155V01	Casper,WY	979947244	25
12172A	San Francisco,CA	979972060	20
15221A	Boston,MA	980378659	20
15560A	Bismarck,ND	980612882	20

IGB — International Gravity Bureau
GSS — Geodetic Survey Squadron

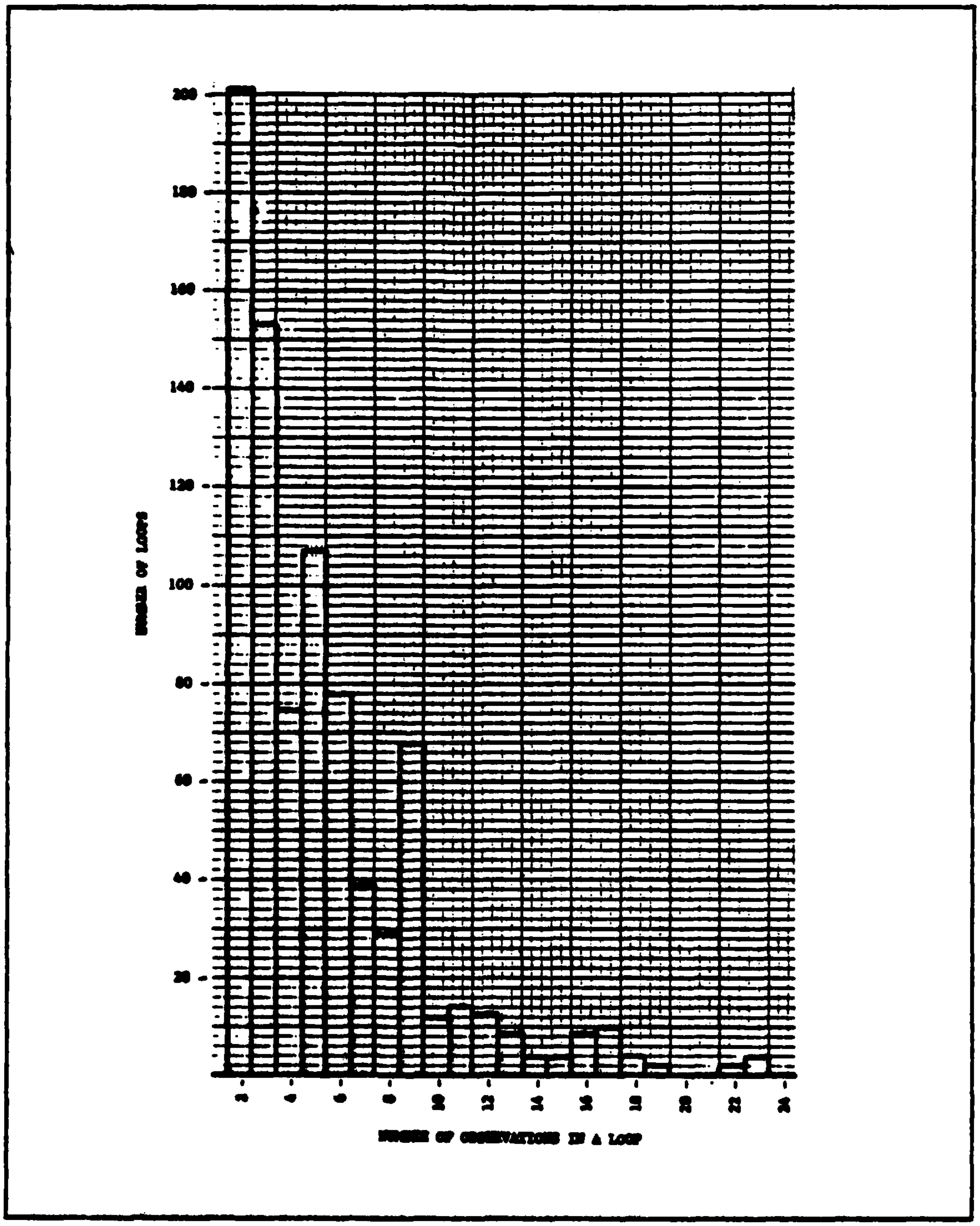


Figure 23 — Histogram of the trips formed and used in the adjustments.

observations. Whenever periodic terms were included, their periods were held fixed and their phase angles were assumed to have a variance of 0.01 radians^2 or ~32.8 degrees^2. The periods were held fixed to eliminate the high correlation which exists between the period and the phase angle in the periodic term. The phase angles were weakly constrained to reflect the accuracy of the estimate of the value of the phase angles determined from the residuals of the observations from ADJUSTMENT A. An estimate of the phase angles for each gravity meter was determined by using a least squares adjustment which used only the residuals associated with the observations from ADJUSTMENT A. The model used was a sinusoidal term having a period of 1206/17 counter units. From these adjustments, the accuracy of the phase angle terms was always less than 0.1 radians. All the accuracies for the phase angles were rounded off to 0.1 radians and used in ADJUSTMENT C.

The initial estimates for the amplitude, phase angle and period of the long wave length sinusoidal term for the gravity meters were based on the analysis of each gravity meter's Calibration Table 1 data. The initial estimates for the amplitude and phase angle of the periodic screw error terms were determined from a least squares fit of the residuals from ADJUSTMENT B for each gravity meter. A summary of the results of the adjustments can be found in Tables 9-12.

Reviewing Table 10, one finds that the aposteriori estimate of the accuracy of the observations for each gravity meter decreased slightly from ADJUSTMENT A values to ADJUSTMENT B except 'G-81' which showed a large change. The reason for this change is unknown at this time and

Table 9 – Summary of the results of ADJUSTMENTS A, B and C.

	ADJUSTMENT A	ADJUSTMENT B	ADJUSTMENT C
Number of Observations	4484	4484	4484
Total Number of Parameters	285	317	351
Number of Gravity Stations	256	256	256
Number of Scale Factor Terms	29	29	29
Number of Periodic Amplitude Terms	0	16	33
Number of Periodic Phase Angle Terms	0	16	33
Number of Weighted Parameters	12	28	45
Number of Trips	837	837	837
Number of Iterations	1	2	2
$V^T PV$	5025.319	3644.095	3433.810
Degrees of Freedom	3374	3358	3341
Aposteriori Variance of Unit Weight	1.48942	1.08520	1.02778

Table 10 — Summary of aposteriori estimates for the accuracy of observations made with the various gravity meters used in ADJUSTMENTS A, B, and C

GRAVITY METER	ADJUSTMENT A	ADJUSTMENT B	ADJUSTMENT C
G-10	19.78	16.78*	17.03*
D-17	17.06	13.76*	13.73*
D-43	30.97	25.85*	25.67*
G-44	16.49	16.38	15.66
G-47	15.11	14.84	15.40
G-50	29.35	29.29	29.32
G-68	31.97	29.71*	28.52*
G-81	31.21	18.58*	18.22*
G-81a	18.72	17.93*	17.67*
G-103	22.03	20.49*	20.20*
G-111	22.64	20.81*	19.56*
G-113	21.94	21.70	21.81
G-115	19.42	17.92*	17.24*
G-115b	21.22	19.59*	19.08*
G-123	21.45	22.42	22.06
G-125	25.47	23.82*	22.61*
G-130	16.82	16.84	16.87
G-131	18.46	17.62*	17.61*
G-140	18.13	18.22	18.18
G-142	23.83	21.56*	21.04*
G-157	23.47	20.29*	20.09*
G-157c	22.74	21.57*	21.33*
G-175	21.76	21.76	21.75
G-176	29.14	27.96	28.01
G-191	18.28	18.06	18.19
G-220	18.09	17.05*	16.62*
G-253	24.56	23.72*	23.62*
G-268	21.96	20.71*	20.01*
G-269	18.58	17.67*	15.60*
Average for All Gravity Meters	22.71	19.73	19.24

a — Calibration Table 1 changed due to addition of electronic readout on 18 October 1977.

b — Calibration Table 1 changed due to replacement of long lever on 27 October 1977.

c — Calibration Table 1 changed due to addition of electronic readout on 30 August 1977.

* — indicates that the standard error is in dial units, all others stanard errors are in μgal.

Table 11 – Summary of the amplitudes of the long wave sinusoidal terms for various gravity meters used in ADJUSTMENTS B and C.

Gravity Meter	Initial Value	Adjustment B Value	Adjustment C Value	Fixed Period
G-10	1624	1576	1584	6000
G-68	3742	3617	3507	11000
G-81	5502	5339	5347	10000
G-81a	417	382	388	5000
G-103	1028	984	1010	7000
G-111	2943	2851	2858	9000
G-115	120	110	103	3000
G-115b	300	267	266	5000
G-125	2174	1860	2182	10000
G-131	410	367	373	5000
G-142	586	551	563	5000
G-157	2760	2646	2638	10000
G-157c	1827	1716	1757	8000
G-220	2158	2041	2075	8000
G-268	4874	4574	4527	11000
G-269	685	758	655	7000

All values are in units of μgal except the periods which are in counter units.

a – Calibration Table 1 changed due to addition of electronic readout on 18 October 1977.

b – Calibration Table 1 changed due to replacement of long lever on 27 October 1977.

c – Calibration Table 1 changed due to addition of electronic readout on 30 August 1977.

Table 12 — Summary of the amplitudes of the periodic screw error terms for various gravity meters used in ADJUSTMENT C.

Gravity Meter	Number of Observations	Initial Value	Adjusted Value
G-10	164	8	7.74 (3.99)
G-68	61	19	18.54 (6.12)
G-81	427	9	8.69 (2.09)
G-81a	171	3	2.40 (4.56)
G-103	125	7	8.28 (3.80)
G-111	427	18	17.22 (2.45)
G-115	369	7	6.95 (2.54)
G-115b	300	5	4.81 (2.19)
G-125	140	13	12.31 (3.10)
G-131	350	4	4.43 (2.39)
G-142	77	9	7.19 (3.87)
G-157	435	4	4.19 (2.54)
G-157c	172	9	8.17 (3.84)
G-220	266	9	7.35 (2.95)
G-253	114	3	5.40 (3.56)
G-268	237	9	10.06 (2.79)
G-269	147	16	18.52 (3.50)

All values are in units of μgal and the values in parentheses represent the estimated aposteriori standard error.

a — Calibration Table 1 changed due to addition of electronic readout on 18 October 1977.

b — Calibration Table 1 changed due to replacement of long lever on 27 October 1977.

c — Calibration Table 1 changed due to addition of electronic readout on 30 August 1977.

was unexpected.

The value of the aposteriori estimate of the accuracies of the observations found in Table 10 is based on the following assumptions. First, the estimated variance of the adjusted observations and their true variance for each gravity meter are nearly equal and will be assumed to be equal. Secondly, the normalized residuals for each gravity meter are assumed to be normally distributed and are effected very little by changes in their assumed observational accuracy. It is known that for a least square adjustment, the variance of the adjusted observations is a function of the variance of the observation and the variance of the residual for the observation. The variance of an adjusted observation is given by

$$\sigma^2_{L_a} = \sigma^2_{L_b} - \sigma^2_{V_i} \tag{6.33}$$

where

$\sigma^2_{L_a}$ — variance of adjusted observations,

$\sigma^2_{L_b}$ — variance of the observations,

$\sigma^2_{V_i}$ — variance of the residuals.

A similar expression exists for the estimate of the variance of an adjusted observation. It is given in the following equation where ^ means the values are estimated quantities derived from the adjustment.

$$\hat{\sigma}^2_{L_a} = \hat{\sigma}^2_{L_b} - \hat{\sigma}^2_{V_i} \tag{6.34}$$

If it were assumed that the true and estimated variance of adjusted observations were equal, then the following expression is obtained by

substituting equation (6.33) into (6.34)

$$\sigma^2_{L_b} = \hat{\sigma}^2_{L_b} + \sigma^2_{V_i} - \hat{\sigma}^2_{V_i} \tag{6.35}$$

The estimated variance of the normalized residuals for a gravity meter is given by

$$\frac{1}{n} \sum \frac{\hat{V}_i^2}{\hat{\sigma}^2_{V_i}} = k \tag{6.36}$$

where V is the residual of the observation i and the value of k is the normalized variance. The expected value of the variance of the true residual of the observations according to Pope [1976] is one. Further, since the value of the residual of an observation is not effected greatly by changes in the variance of that observation, one would the true residual of an observation, V_i, to be nearly equal to the estimated residual of the observation, $\hat{V}_i$. In which case, the variance of the true residuals is related to the estimated variance of the estimated residuals by

$$\sigma^2_{V_i} = k\hat{\sigma}^2_{V_i} \tag{6.37}$$

By substituting equations (6.36) and (6.37) into (6.35) results in the relationship used to obtain the aposteriori estimate of the accuracy of the observations found in Table 10.

$$\sigma^2_{L_b} = \hat{\sigma}^2_{L_b} + \left(\frac{1}{n}\sum\frac{\hat{V}^2_i}{\hat{\sigma}^2_{V_i}} - 1 \right)\hat{\sigma}^2_{V_i} \tag{6.38}$$

Since there was no reliable information about the accuracy of the observations made with the LaCoste & Romberg gravity meters, an accuracy of an observation had to be estimated. The value selected was 0.02 counter units which is approximately equivalent to 20 μgal. This value was based on the results from preliminary adjustments which tested various estimates for the observational accuracy. The aposteriori estimate of the accuracy of the observations from these adjustments indicated accuracies around 0.02 counter units for each gravity meter would be appropriate. Using the relationship expressed in equation (6.38), the values found in Table 10 were computed after the final adjustments had been done. These values indicate that the gravity meters used in this study had approximately the same observational accuracy. It is in the range of 0.015 to 0.025 counter units which is equivalent to an accuracy of about 15 to 25 μgal. counter units.

To determine if there is a significant difference between the adjustment model used, it would be convenient if a statistical test existed to test the significance of the nonlinear models used. Unfortunately, the tests proposed for testing nonlinear hypotheses are often too complex to be used in practice and depend on the nonlinearity of the problem [Hamilton, 1964, p 157]. In practice, tests for the linear case are performed. Considering that the tests will be inexact,

a greater significance level for the tests might be warranted.

Assuming the linear model for the two adjustments, ADJUSTMENT I and ADJUSTMENT II, used the same set of observations, then one can test if a set of parameters, Q, has any significant effect on the adjustment by performing the adjustment with and without the Q-parameters and form a test statistic, F, based on the degrees of freedom of the adjustments, the number of additional constraints, and the values of the V^tPV.

Let ADJUSTMENT I be performed including the Q-parameters and ADJUSTMENT II be performed excluding the Q-parameters. Then the null hypothesis that all Q-parameters could be set to zero is rejected if

$$F = \frac{V^tPV_{II} - V^tPV_I}{V^tPV_I} \frac{r}{s} > F_{s,r,1-\alpha} \qquad (6.39)$$

where r is the degree of freedom for ADJUSTMENT I and s is the number of Q-parameters.

Using this concept, a test was performed to see if there was a significant difference between ADJUSTMENT B and ADJUSTMENT C at the 5% significance level. This test would indicate if the periodic screw error terms included in ADJUSTMENT C were significant. From information contained in Table 8, the test statistic, F, was calculated based on V^tPV_I = 3433.810, V^tPV_{II} = 3644.095, r = 3341 and s = 34 which results in F = 6.02 with $F_{34,3341,0.95} = 1.43$. Therefore, at the 5% significance level, the null hypothesis that the periodic screw error terms are not significant is rejected.

Unfortunately, there appears to be no known statistical test which can be used to test ADJUSTMENT A against ADJUSTMENT B or ADJUSTMENT C

because ADJUSTMENT A involves a nonlinear relationship of the true observables, counter readings.

For very large degrees of freedom as in these adjustment, F-tables are generally not available, however, a fairly good approximation to the F-distribution percentile when the degrees of freedom are larger than 30 according to Kossack and Henschke [1975] can be obtained from

$$\log F_{\alpha}(\nu_1,\nu_2) \simeq \left(\frac{a}{\sqrt{h-b}}\right) - cg \tag{6.40}$$

where $h = 2\nu_1\nu_2/(\nu_1+\nu_2)$ and $g = (\nu_2 - \nu_1)/\nu_1\nu_2$ with the values of a, b and c being a function of α . See Table 13 for some selected values. It should be noted that ν_1 and ν_2 represent the degrees of freedom of the numerator and denominator respectively.

It is clear that, whenever possible, the original counter readings should be used as the observable in the model for a gravity base station network adjustment. One possible way of accomplishing this is to model the Calibration Table 1 information by means of a long wave sinusoidal term. The use of such a model requires the observations be made over a sufficiently large range and be adequately distributed throughout that range. In addition, the periodic screw error terms should also be include in the model provided it can be justified by the distribution of the data. In order to check whether a periodic screw error term for a particular gravity meter should be included in the model, graphs similar to those in Figure 24 and Figure 25 were produced for each possible period of the periodic screw error term. If the distribution of the observations over the period selected was lacking,

Table 13 — Values of the parameters used in the approximation of the F-distribution for large degrees of freedom.

Value of α	Value of a	Value of b	Value of c
0.50	0.0	——	290.000
0.75	0.5859	0.58	0.355
0.90	1.1131	0.77	0.527
0.95	1.4287	0.95	0.681
0.975	1.7023	1.14	0.846
0.99	2.0206	1.40	1.073
0.995	2.2373	1.61	1.250
0.999	2.6841	2.09	1.672
0.9995	2.8580	2.30	1.857

as is the case in Figure 25, then including the periodic screw error term for that period in the model might not be appropriate. However, if the distribution is similar to that given in Figure 24, then it would be appropriate to include a periodic screw error term for that period in the model for that instrument.

6.4 Consistency of Absolute Gravity Values

As can be seen in Table 4 and Table 5, see section 4.2, some of the values of gravity determined by the Italians, Marson and Alasia, differ considerably from the value determined by Hammond at the same site. The reason for these differences is not known at this time but is presently being investigated by Hammond. To show the effect on the station values of the control selected for a gravity base station network adjustment, three additional adjustments, ADJUSTMENT D, ADJUSTMENT E, and ADJUSTMENT F, were performed using ADJUSTMENT C'S model. ADJUSTMENTS D, E and F used the same model but with difference absolute stations constrained. In ADJUSTMENT D, the absolute stations constrained were those determined by Hammond. In ADJUSTMENT E, the absolute stations determined by the Italians, Marson and Alasia, were constrained. In ADJUSTMENT F, only four absolute stations were constrained. The four stations selected to be constrained were the ones whose determined values were in good agreement with both Hammond's and the Italian's values. A summary of the adjusted absolute station values can be found tn Table 14. A summary of how the adjusted absolute station values differed from their initial values for

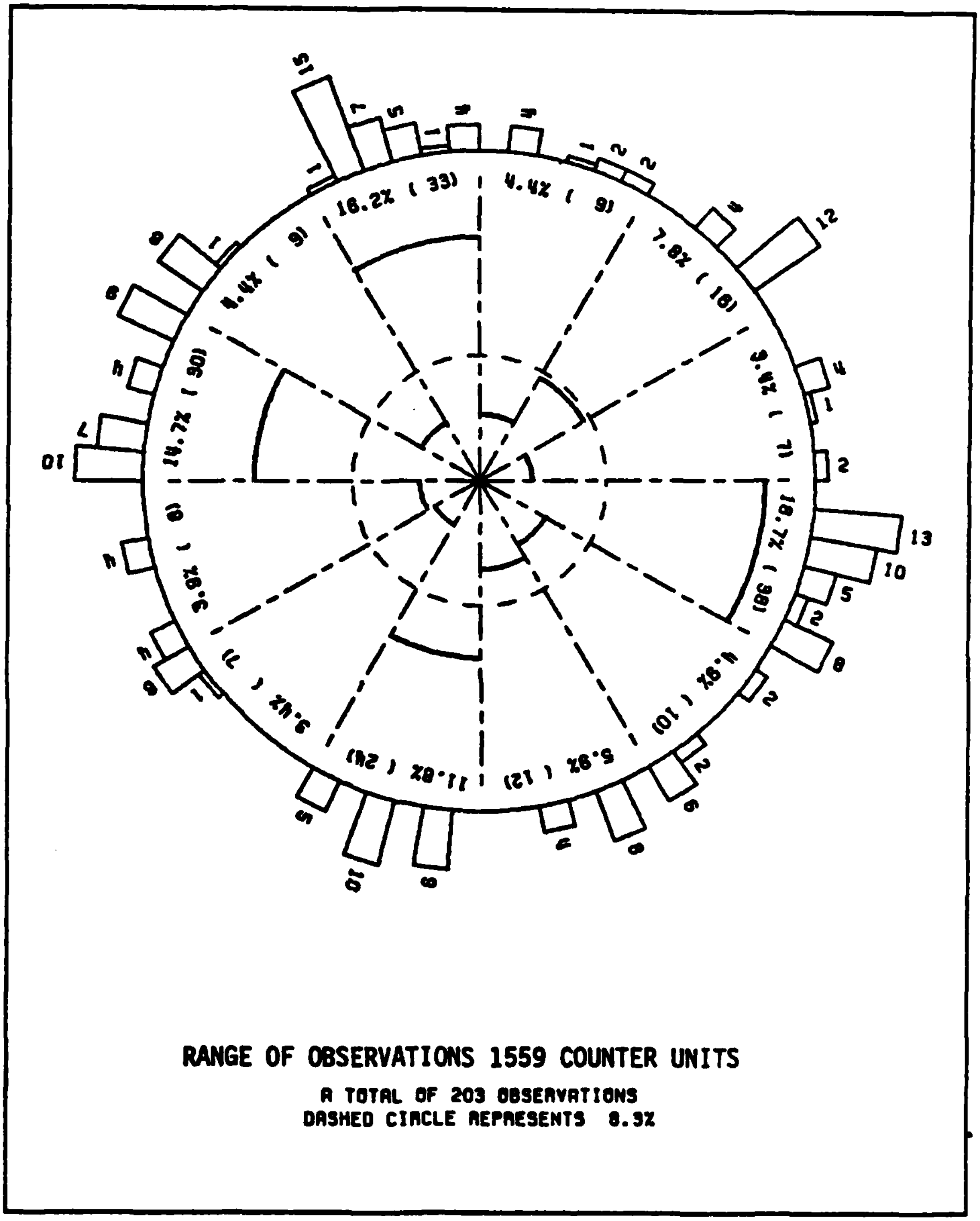
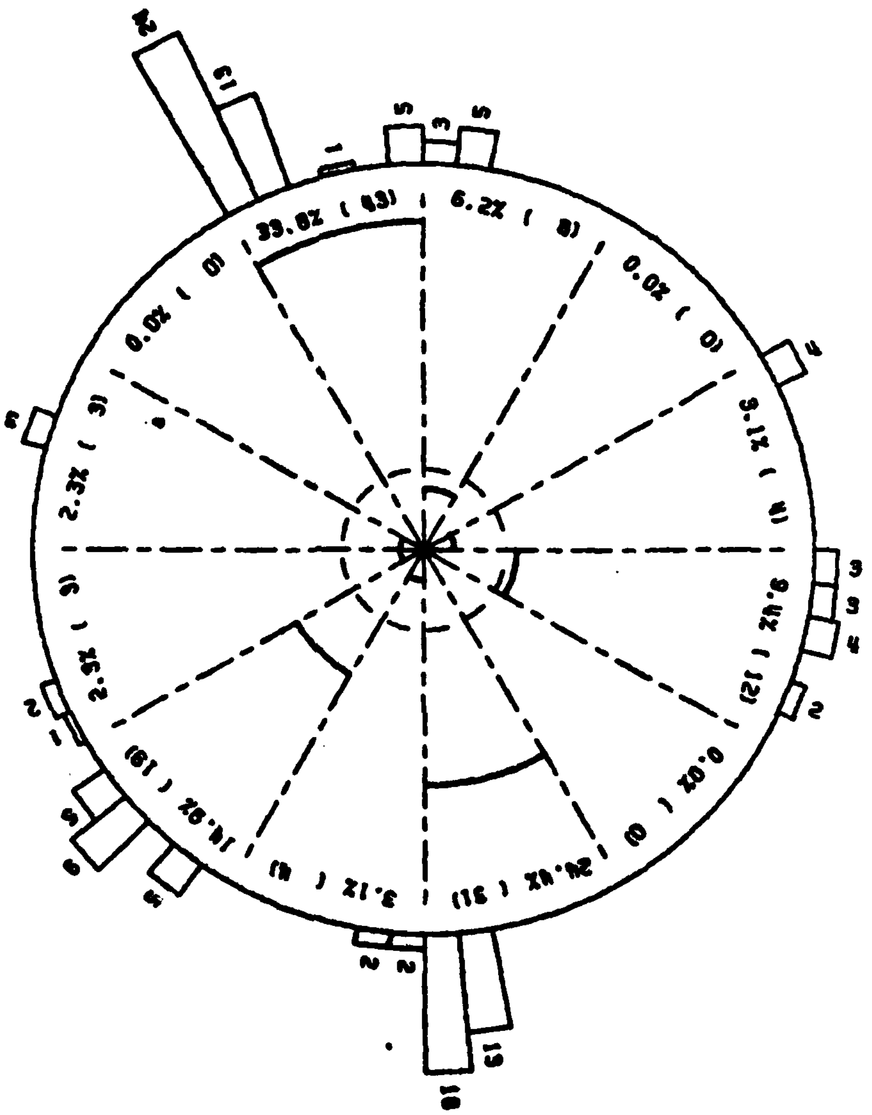


Figure 24 — Distribution of a set of observations for G-131 assuming a period of 1206/17 counter units — set no. 1



RANGE OF OBSERVATIONS 1317 COUNTER UNITS

A TOTAL OF 127 OBSERVATIONS
DASHED CIRCLE REPRESENTS 8.3%

Figure 25 — Distribution of a set of observations for G-131 assuming a period of 1206/17 counter units — set no. 2

Table 14 – List of the values the absolute stations used and the results of ADJUSTMENTS D, E, and F.

IBG/GSS Code	Station Name	ADJ. D Value Used	ADJ. E Value Used	ADJ. F Value Used
119S04	McDonald Obs.	978820087	978820097	978820092b
119S03	McDonald Obs.	978828655	———	———
08150C	Miami	———	979004319	———
11926A	Holloman	979139600	979139584	979139592b
119C03	Mt. Evans	979256059	———	———
119C01	Trinidad	979330382b	———	———
11994H	Denver	979598277	979598267	979598272b
15505D	Boulder	979608601	979608498	———
155V01	Casper	979947244a	———	———
12172A	San Francisco	———	979972060	———
155V03	Sheridan	980208938b	980209007	———
15221A	Boston	980378673	980378659	980378666b
156E05	Great Falls	980497339	980497412	———
15560A	Bismarck	———	980612882	———
Number of Observations		4484	4484	4484
Number of Parameters		351	351	351
Number of Weighted Parameters		44	43	37
Number of Trips		837	837	837
Number of Iterations		2	2	2
V PV		3517.475	3578.143	3337.026
Degrees of Freedom		3340	3339	3333
Aposteriori Variance of Unit Weight		1.05314	1.07162	1.00121

IGB – International Gravity Bureau
GSS – Geodetic Survey Squadron

NOTE: The variance for each gravity value is assumed to be 400 μgal except those marked with an 'a' which have a variance of 625 μgal and those marked with a 'b' which have a variance of 200 μgal . The values of gravity are given in units of μgal.

Table 15 – Summary of the difference between the adjusted absolute station values used as control in the various adjustments and their initial values.

IBG/GSS Code	Station Name	The value in μgal to be added to the initial station value to arrive at the adjusted station values		
		ADJ. A	ADJ. B	ADJ. C
119S04	McDonald Obs.	–31	–7	–7
119S03	McDonald Obs.	–30	–13	–15
08150C	Miami	18	4	4
11926A	Holloman	–2	7	4
119C03	Mt. Evans	40	18	21
119C01	Trinidad	–27	–31	–26
11994H	Denver	35	29	27
15505D	Boulder	29	21	20
155V01	Casper	–22	–27	–26
12172A	San Francisco	1	–5	–7
15221A	Boston	–11	–6	–6
15560A	Bismarck	–14	0	0

IBG/GSS Code	Station Name	ADJ. D	ADJ. E	ADJ. F
119S04	McDonald Obs.	–1	–29	–4
119S03	McDonald Obs.	–14	——	——
08150C	Miami	——	3	——
11926A	Holloman	11	–9	–3
119C03	Mt. Evans	29	——	——
119C01	Trinidad	–21	——	——
11994H	Denver	55	36	12
15505D	Boulder	–42	24	——
155V01	Casper	9	——	——
12172A	San Francisco	——	21	——
155V03	Sheridan	14	–23	——
15221A	Boston	1	4	–5
156E05	Great Falls	–15	–33	——
15560A	Bismarck	——	8	——

IGB – International Gravity Bureau
GSS – Geodetic Survey Squadron

NOTE: The initial station values for ADJ. A, ADJ. B, and ADJ. C can be found in Table 8. The initial station values for ADJ. D, ADJ. E, and ADJ. F can be found in Table 14. Values are given for only those stations that were used as control in the adjustments.

ADJUSTMENTS A, B, C, D, E, and F can be seen in Table 15.

Tests similar to the ones used to compare ADJUSTMENTS B and C were used to compare ADJUSTMENTS D and F and to compare ADJUSTMENTS E and F. The tests were used to indicate if the added constraints introduced by the additional absolute sites, were consistent with the four common absolute sites used in ADJUSTMENT F. Using the information in Table 14, at the 5% significance level, the test statistics, F1, for comparison of ADJUSTMENT D and F and the test statistic, F2, for comparison of ADJUSTMENT E and F, were computed. The values obtained were F1 = 25.747 and F2 = 40.138 with $F_{7,3333,0.95} \simeq 2.0$ and $F_{6,3333,0.95} \simeq 2.1$. These results indicate that the additional absolute sites constrained in ADJUSTMENT D and E were not consistent with the four absolute sites constrained in ADJUSTMENT F.

The effect on the adjusted station values becomes apparent when the difference between the three adjustments are compared. Figure 26 shows the difference in the station values of ADJUSTMENT E – ADJUSTMENT D. Similarly, Figure 27 and Figure 28 shows the difference in station value of ADJUSTMENT D – ADJUSTMENT F and ADJUSTMENT E – ADJUSTMENT F respectively. It is apparent from Figures 26–28, that some sort of linear trend with respect to the value of gravity exist for these differences. The linear trend could be caused by some nonlinear scale factor relationship for the gravity meter which is a function of the gravity at the station or, more likely, by an inconsistent set of absolute station values. As with any gravity base station network, any error in the value of the absolute stations is absorbed directly into

he scale factor terms for the gravity meters used. The detection of ad or inconsistent absolute values is very difficult, especially when here might be more than one errant station value. For example, as can e seen in Figure 26, Miami and Bsimarck, two stations whose values ere determined only by the Italians, appear to be inconsistent with he other absolute sites. But from the information in Table 15 for DJUSTMENT E, these two stations appear to be consistent with the other tations determined by the Italians. The apparent inconsistency of the bsolute determinations makes it very difficult to check how well arious mathematical models for the gravity meter behavior perform ecause the control for the gravity network is provided by the nconsistent absolute gravity station values.

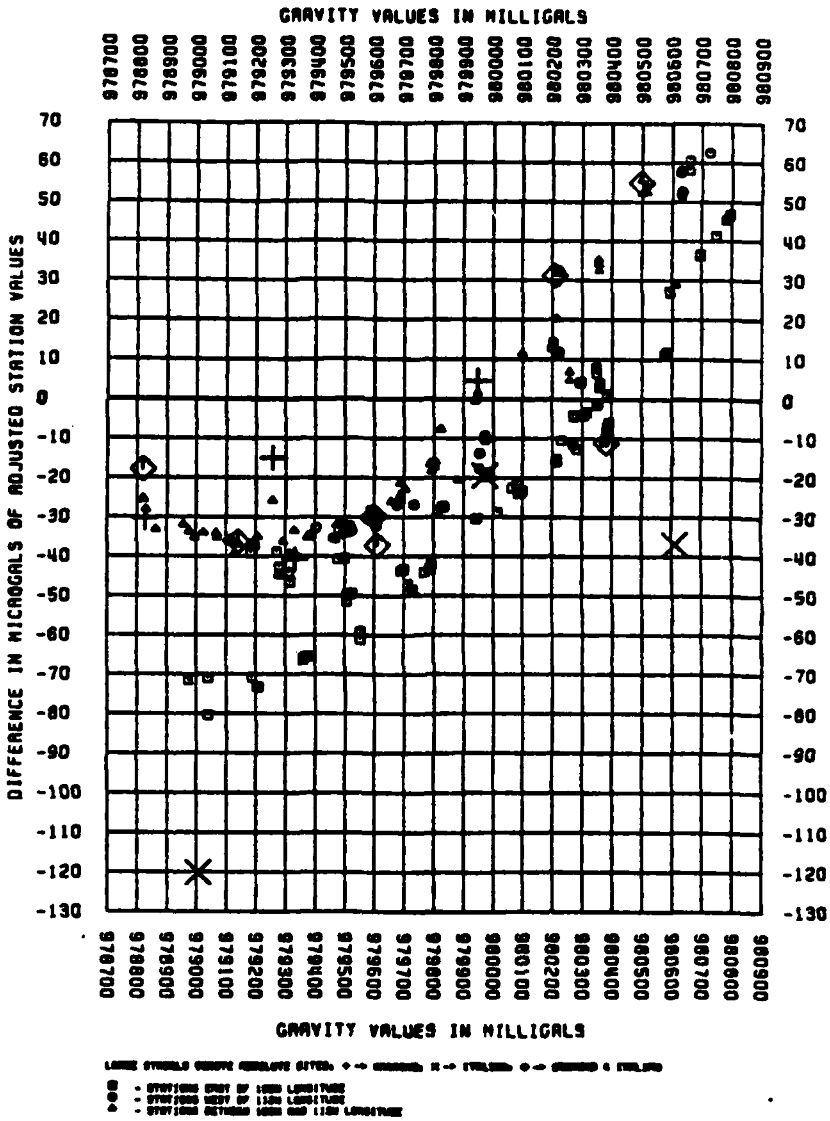


Figure 26 — Plot of the difference between adjusted station values for ADJUSTMENT E — ADJUSTMENT D

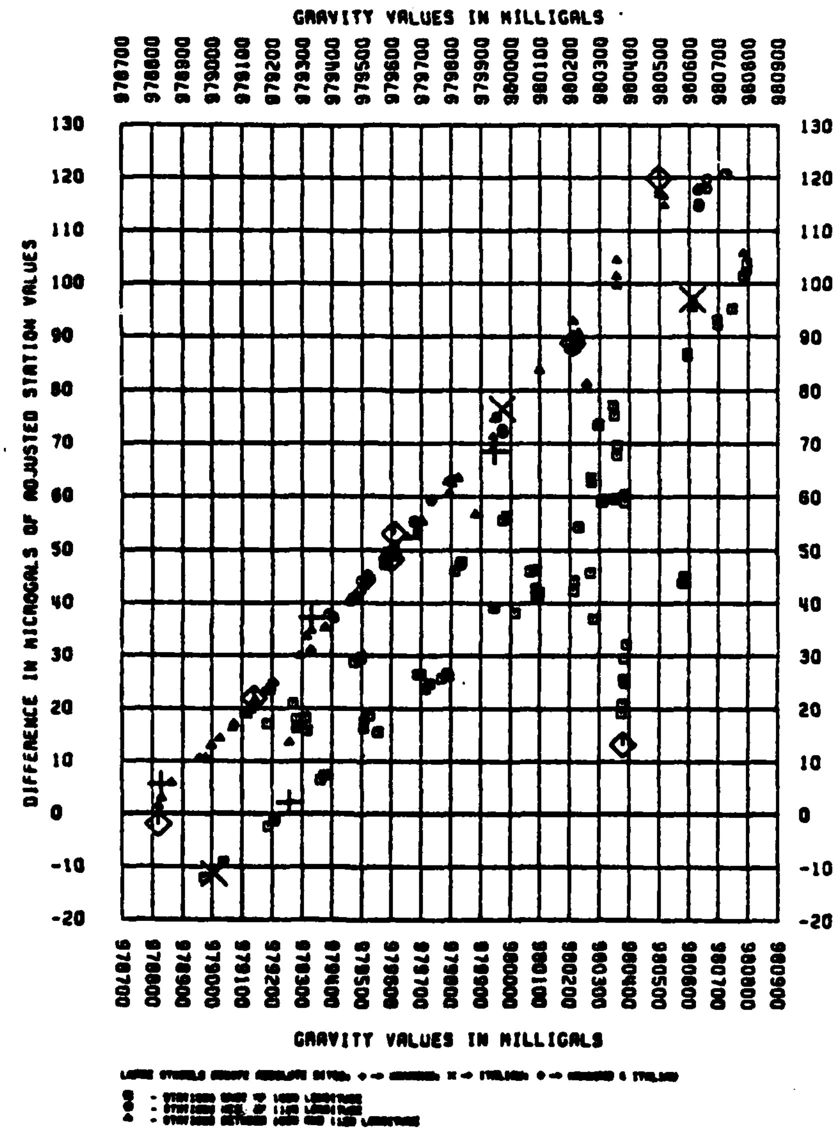


Figure 27 — Plot of the difference between adjusted station values for ADJUSTMENT D — ADJUSTMENT F

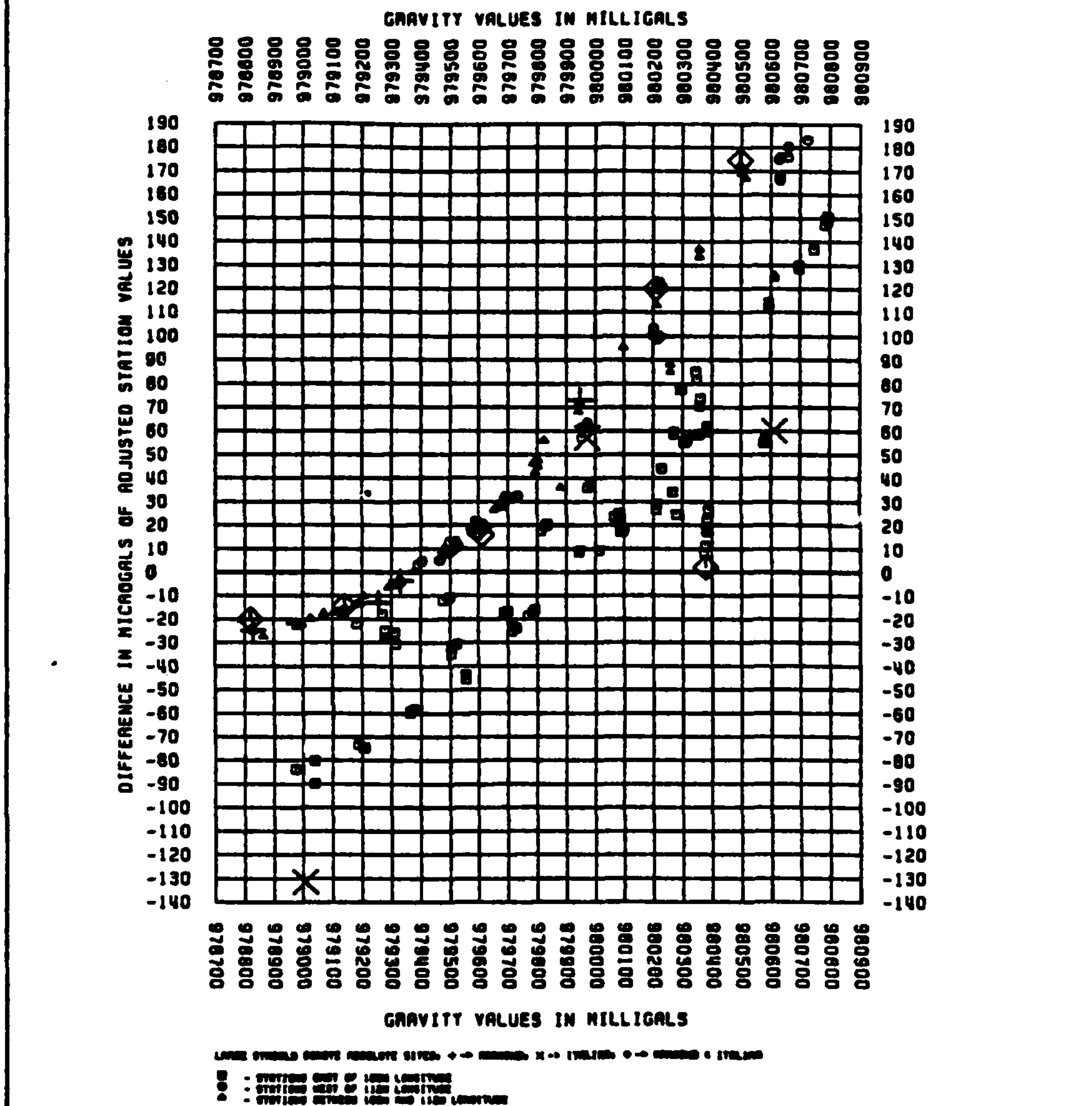


Figure 28 — Plot of the difference between adjusted station values for ADJUSTMENT E — ADJUSTMENT F

CHAPTER SEVEN

CONCLUSIONS

To obtain the value of gravity for stations in a gravity base station network established by gravity ties made with LaCoste & Romberg 'G' gravity meters requires that an adequate model be produced that can be used to represent the gravity meter's behavior. For networks having a sufficiently large number of stations spanning a wide range of gravity values, such as the United States Gravity Base Station Network which includes the Mid-Continent Calibration Line, a model has been developed which will permit the actual counter readings to be used directly as the observables. This model is based on the ability to approximate the gravity meter's Calibration Table 1 information by means of a first order polynomial term plus sinusoidal terms. The first order polynomial term and long wave length sinusoidal term, when determined for the range of the observations made for the United States Gravity Base Station Network, resulted in a RMS of the least square fit to the gravity meter Calibration Table 1's values in milligal of better than 10 μgal for most of the gravity meters used in the network. The computer program used in the adjustments was developed so a linear drift rate term could be included in the mathematical model. However, since physical explanation for the drift in a gravity meter is a series

of small tares occuring at random times, the drift term was not included in the adjustment model used.

With this type of model, including additional sinusoidal terms is relatively easy. These additional terms are used to represent any periodic screw error effect that might be present in a gravity meter. Due to the construction of the gravity meter's gear box, there is the possiblity that sinusoidal terms having periods of 1206, 1206/17, 134/17 and 1 counter units could exist for gravity meters having the old gear box. For those gravity meters having the new gear box, periods of 220, 220/3, 22/3 and 1 counter units are possible. In addition, periods of half those values are also possible. However, it became apparent that the most likely cause of a periodic screw error effect in the instrument results from the contact between the jewel press fit in the measuring screw and the spherical metal ball at the end of the lever linkage assembly. If the fit between these two parts is not perfect, a type of periodic effect could easily be introduced. The period of this effect would be 1206/17 or 220/3 counter units depending on which gear box was installed in the gravity meter.

This study indicated that when a periodic term having a period of 1206/17 counter units was included in the model for the gravity meter, amplitudes as large as 18 μgal were found. However, most amplitudes were below 10 μgal. This is far less than the values predicted by Harrison and LaCoste [1978] of 35 to 50 μgal. This does not mean that periodic screw error effects as large as 35 to 50 μgal do not exist. It only means that of the 14 different gravity meters used in this

study for which there appeared to be sufficient observations to permit solving for the periodic screw error term, none exhibited any large periodic screw error amplitudes.

This study also indicated that the behavior of the gravity meters appears to be quite stable with respect to time, changing only when there was an actual change in the components of the meter, such as when the long lever is replaced. It cannot be concluded that the linear scale factor changes from year to year as has been hypothesized since there is no evidence of this occurring. However, the selection of which absolute sites are used as the control for the network has a great influence on the value of the linear scale factor determined. This is because an error in the value of any absolute station propagates almost directly into the determination of the linear scale factor term. There is no substitute for good control in a gravity base station network.

This study indicates that the estimated accuracy of observations made with a LaCoste & Romberg 'G' gravity meter is around 0.02 counter units. The accuracy of the absolute station determinations used in this study is probably closer to 20 μgal than to their reported accuracy of around 10 μgal.

For existing gravity base station networks for which a variance—covariance matrix for the station values exist, an algorithm was developed which could be used to determine the gravity tie that should be made which would improve the network the most in the sense of minimizing the trace of the resulting variance—covariance matrix of the

gravity stations used in the adjustment. This algorithm was not applied to the United States Gravity Base Station Network because the accuracy of the absolute sites which were used as control for the adjustment did not appear to be consistent with each other.

The problem of matching station observations with their proper station site description is real and at times very confusing. The system adopted for identifying gravity stations, if not uniformly followed by the organizations collecting gravity meter data, will result in needless errors and confusion. The actual system use is not that critical as long as each station is assigned only one unique identification code. But, it must be used by everyone.

Another problem which should be eliminated is that whenever a change is made in a station's site description form, the change should be noted with at least the date that the change was made. Otherwise, old and possibly erroneous information could be used.

This study leaves many areas for future studies. The understanding of how the behavior of the LaCoste & Romberg 'G' gravity meter could be modelled as the range of the observations gets larger needs to be investigated. If the Calibration Table 1 information cannot be adequately modelled with a single sinusoidal term for a large range, then the use of some type of piece wise continuous curve could be investigated. Methods are needed that will permit during the calibration of the gravity meter detection of periodic screw error effects. This would involve investigating periods other than 1206/17 counter units. However, only periods which have some physical reason

for existing should be considered.

The modelling of the other possible systematic effects caused by such things as changes in the distribution of the mass of the atmosphere, changes in the groundwater level, and possibility of instrumental drift needs to be studied further. However, the backbone of any future studies rests on the ability to obtain adequate control for the gravity base station network. A set of consistent absolute station gravity values is essential; otherwise, wrong conclusions will almost surely result.

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APPENDIX A

Formation Of Weight Matrix For A Trip

In the formation of the weight matrix for the gravity observation equations, time can be saved in the formation of this matrix by recognizing the pattern on the matrices used to form it. For each trip, the weight matrix, M^{-1}, needs to be formed from the matrix of partials with respect to the observation, B, and the covariance matrix of the observations, $\sum_{L_b}$, where

$$M = B\textstyle\sum_{L_b} B^t \tag{A.1}$$

$$M^{-1} = (B\textstyle\sum_{L_b} B^t)^{-1} \tag{A.2}$$

and the structure of B has the form

$${}_{n-1}B_n = \begin{bmatrix} b_1 & -b_2 & & & & \\ & b_2 & -b_3 & & \bigcirc & \\ & & & \ddots & & \\ & \bigcirc & & b_{n-2} & -b_{n-1} & \\ & & & & b_{n-1} & -b_n \end{bmatrix} \tag{A.3}$$

where n is the number of observations and b_i is the magnitude of the partial with respect to the observation, i.

Assuming that the observations made with an instrument are independent and have the same accuracy, then $\sum_{L_b}$ can be written as

$$\Sigma_{L_b} = k_n I_n \tag{A.4}$$

where

k — is a scalar and has the value of the variance of each observation,

I — is an nxn identity matrix.

Rewriting equation (A.1) as

$$M = k(BB^t) \tag{A.5}$$

where the structure of BB^t has the form

$$BB^t = \begin{bmatrix} b_1^2+b_2^2 & -b_2^2 & & & & \\ -b_2^2 & b_2^2+b_3^2 & -b_3^2 & & 0 & \\ & & \ddots & & & \\ & & & -b_{n-2}^2 & b_{n-2}^2+b_{n-1}^2 & -b_{n-1}^2 \\ & 0 & & & -b_{n-1}^2 & b_{n-1}^2+b_n^2 \end{bmatrix} \tag{A.6}$$

when both sides of equation (A.5) is inverted, M^{-1} becomes

$$M^{-1} = \frac{1}{k} (BB^t)^{-1} \tag{A.7}$$

If the partials with respect to the observations all have the same magnitude L, then M can be written as

$$M = kL^2Q \tag{A.8}$$

with the structure of Q being

$$Q = \begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & 0 & \\ & & \ddots & & & \\ & 0 & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix} \tag{A.9}$$

An analytical inverse exists for equation (A.9) [Gregory and Karney, 1968, pp 45–46] of the form

$$Q^{-1} = \frac{1}{n + 1} C \tag{A.10}$$

where n is the order of Q and the elements of C are given by the relationship

$$C_{ij} = C_{ji} = i(n - j + 1) \qquad i \le j \tag{A.11}$$

Thus M^{-1} has the form

$$M^{-1} = \frac{1}{(n + 1)kL^2} C \tag{A.12}$$

APPENDIX B

Formation Of The Normal Equations

For efficiency, the contribution to the normal equations is based on performing only non-zero multiplication. The non-zero partials with respect to the parameters are stored on a file that can be sorted with a 16 byte structure like

BYTES	VALUE	TYPE
1- 8	V - non-zero partial with respect to the parameters	floating point number
9-10	R - equation number within a trip of the partial	integer
11-12	C parameter number associated with the partial	integer
13-14	T - trip number	integer
15-16	S - sequence number of partial within a trip	integer

The misclosure vector, W, is stored sequentially in an array in trip number, equation number order.

The file of non-zero partials are then sorted on T, C and R in ascending order. With the non-zero partials in this order, the non-zero elements of can be formed efficiently and each element formed is saved on another sortable file with the same structure desribed above. Since the structure of M^{-1} is a block diagonal matrix with a

block for each trip, the non-zero contribution to A^tM^{-1} needs to be computed only for each trip where i is the trip number.

To compute $A^tM^{-1}A$ and $A^tM^{-1}W$ efficiently, all non-zero elements of A^tM^{-1} are sorted on R, T and C in ascending order and all non-zero elements of A are sorted on C, T and R in ascending order. Then if row by row, a row of non-zero elements A^tM^{-1} were brought into core, the contribution to the corresponding row of the normal matrix, N, could be computed from the non-zero elements of A. N is the relationship $A^tM^{-1}A$. As each row's contribution was completed, it would be saved. At the same time, the contribution to U, which is the relationship $A^tM^{-1}W$, would be computed.

Since the normal matrix, N, is stored in a sub-block form, a block of rows of the non-zero elements of A^tM^{-1} are brought into core and a block of rows of N and U are formed and saved. Thus the largest array required would have (number of rows in a block) times (number of parameters).

APPENDIX C

Computation Of The Variance Of Residuals

The computation of the covariance matrix of the residuals requires a huge amount of computational effort. However, if only certain covariances of the residuals are desired, the computational effort required can be considerably reduced by taking advantage of the pattern and sparse matrices involved.

For the combined mathematical model $F(X_a, L_a) = 0$, it can be shown that from its linearized form

$$AX + BV + W = 0 \tag{C.1}$$

that the least square solution gives the residual vector as

$$V = - P^{-1}B^tM^{-1}(AX + W) \tag{C.2}$$

where

$$M^{-1} = (B\Sigma_{L_b}B^t)^{-1} \tag{C.3}$$

$$X = - (A^tM^{-1}A)^{-1}A^tM^{-1}W \tag{C.4}$$

By substituting equations (C.3) and (C.4) into equation (C.2) gives

$$V = P^{-1}B^tM^{-1}(\ I - A(A^tM^{-1}A)^{-1}A^tM^{-1})W \tag{C.5}$$

The variance and covariance propagation for a linear function given in equation (C.5) expressed as

$$V = G(W) \tag{C.6}$$

results in the covariance matrix for the residuals of

$$Q_V = (\ \frac{\partial G}{\partial W}\ )\ Q_W\ (\ \frac{\partial G}{\partial W}\ )^t \tag{C.7}$$

where

$$\frac{\partial G}{\partial W} = P^{-1}B^tM^{-1}(\ I - A(A^tM^{-1}A)^{-1}A^tM^{-1}) \tag{C.8}$$

$$Q_W = BP^{-1}B^t = M \tag{C.9}$$

Substituting equations (C.8) and (C.9) into equation (C.7) yields

$$Q_V = P^{-1}B^t(M^{-1} - M^{-1}A(A^tM^{-1}A)^{-1}A^tM^{-1})BP^{-1} \tag{C.10}$$

letting $\overline{M} = M^{-1} - M^{-1}A(A^tM^{-1}A)^{-1}A^tM^{-1}$ then equation (C.10) can be written as

$$Q_V = P^{-1}B^t\ \overline{M}\ B\ P^{-1} \tag{C.11}$$

Exploiting the fact that P^{-1} is a diagonal matrix and the B matrix is sparse and has a definite pattern, it can be shown that the covariance for a residual is given by one of the following relationships:

$$\sigma^2_{V_{jj}} = k_1 b_j^2 (\bar{m}_{jj} + \bar{m}_{j-1,j-1} - 2\,\bar{m}_{j-1,j}) \qquad 1<j<n \tag{C.12}$$

$$\sigma^2_{V_{11}} = k_1 b_1^2\, \bar{m}_{11} \tag{C.13}$$

$$\sigma^2_{V_{nn}} = k_1 b_n^2\, \bar{m}_{n-1,n-1} \tag{C.14}$$

$$\sigma^2_{V_{1j}} = -\, k_1 b_1 b_j (\bar{m}_{1j} - \bar{m}_{1,j-1}) \tag{C.15}$$

$$\sigma^2_{V_{ij}} = k_1 b_i b_j (\bar{m}_{ij} + \bar{m}_{i-1,j-1} - \bar{m}_{ii} - \bar{m}_{i-1,j}) \qquad \begin{array}{c} i \neq j \\ 1<i<n \\ 1<j<n \end{array} \tag{C.16}$$

$$\sigma^2_{V_{1n}} = k_1 b_1 b_n\, \bar{m}_{1,n-1} \tag{C.17}$$

$$\sigma^2_{V_{jn}} = k_1 b_j b_n (\bar{m}_{j-1,n-1} - \bar{m}_{j,n-1}) \qquad 1<j<n \tag{C.18}$$

where

k — is the variance of the observations for a trip.

n — is the number of observations for the trip.

$\bar{m}$ — is the sub-block of $\bar{M}$ associated with the trip.

The $\bar{M}$ matrix can be partitioned as follows

$$\bar{M} = \left[\begin{array}{c:c:c:c:c} \bar{M}_1 & & & & \\ \hdashline & \bar{M}_2 & & & \\ \hdashline & & \ddots & & \\ \hdashline & & & \bar{M}_{t-1} & \\ \hdashline & & & & \bar{M}_t \end{array}\right] \tag{C.19}$$

where l is the number of equations formed for a trip.

If the variance of the residuals is desired, then only the tridiagonal elements of $\bar{M}$ are needed which cuts down on the amount of computation required.

How to obtain the tridiagonal elements of $\bar{M}$ will now be discussed. Remembering that M is a block diagonal matrix with one block for each trip and the size of each block is equal to the number of equations formed for the trip and that $\bar{M} = M^{-1} - M^{-1}A(A^tM^{-1}A)^{-1}A^tM^{-1}$, the major effort is in computing the tridiagonal contribution to $\bar{M}$ of $M^{-1}A(A^tM^{-1}A)^{-1}A^tM^{-1}$. Since both $(A^tM^{-1}A)^{-1}$ and the non-zero elements of A^tM^{-1} are available from the adjustment, it becomes a matter of what is the best way of obtaining the tridiagonal contribution.

By sorting the non-zero elements of A^tM^{-1} on their columns and rows in ascending order, the multiplication required to form the elements of $(A^tM^{-1}A)^{-1}A^tM^{-1}$ can most efficiently be done. Each element's value along with its row and column in $(A^tM^{-1}A)^{-1}A^tM^{-1}$ are saved on a sortable file and then sorted in column, row ascending order. To compute the required tridiagonal elements, the appropriate columns of $M^{-1}A$ and $(A^tM^{-1}A)^{-1}A^tM^{-1}$ are brought into core, the multiplication

performed and the appropriate tridiagonal elements are subtracted from M^{-1} and the variance of the residual is computed.